

LEARNING GUIDE

MATHEMATICS

DBE

SOLUTIONS

2nd EDITION

MTH-5173-2

SCI

GEOMETRIC
REPRESENTATION

IN A FUNDAMENTAL CONTEXT 2

IN COMPLIANCE
WITH THE NEW
PROGRAM
OF STUDY

SOFAD

LEARNING GUIDE

MATHEMATICS

FBD

SOLUTIONS

MTH-5173-2

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GEOMETRIC
REPRESENTATION
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CONTEXT 2

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SOFAD

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STRUCTURE OF THE LEARNING GUIDE

Welcome to the learning guide for the **Geometric Representation in a Fundamental Context 2** course. The aim of this course, which is the third in the **Secondary V Science** sequence, is to develop your ability to deal with situations that involve spatial descriptions or representations. To achieve this, you will learn about geometric loci, in particular:

- the circle
- the ellipse
- the hyperbola
- the parabola.

You will also be introduced to the concept of vectors and their properties and deepen your knowledge of trigonometric identities.

You will be required to use various solution strategies to understand and model situational problems. You will need to use your mathematical reasoning skills. You will also have to describe how you solved these problems clearly and thoroughly using mathematical language.

You are now invited to complete the learning activities found in the three chapters of this guide and to enrich your knowledge of geometry.

Portailsofad.com

Go to portailsofad.com for videos, ICT activities and printable versions of resources that are complementary to the SOLUTIONS series, which you can use throughout your learning journey.

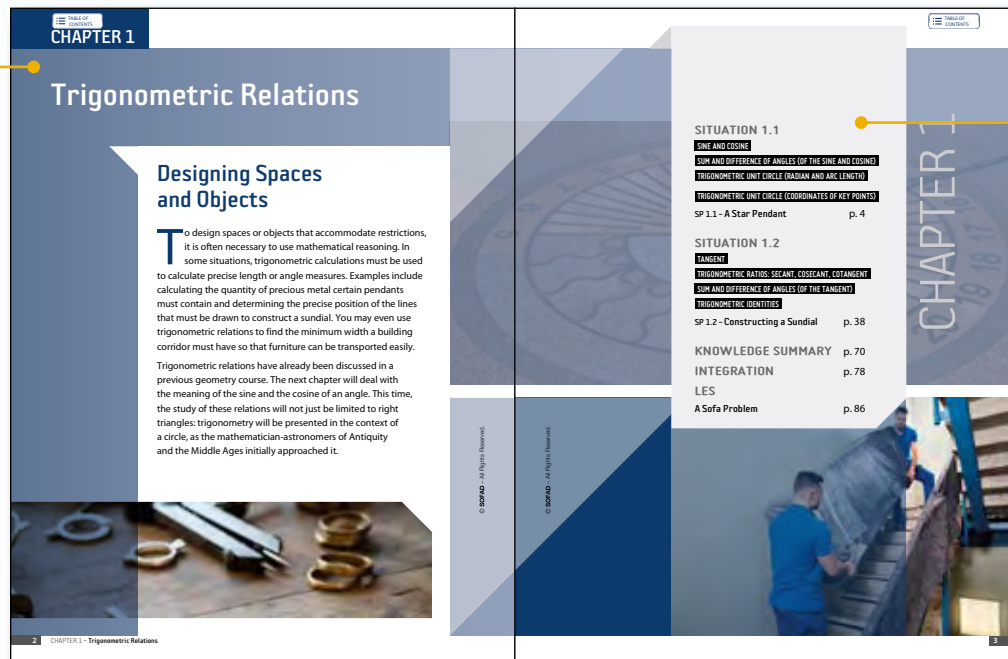


CHAPTER COMPONENTS

The learning process followed in each chapter enables students to progress by building on what they have learned from one section to the next. The following diagrams illustrate this approach and specify the pedagogical intent of each section.

CHAPTER OPENING

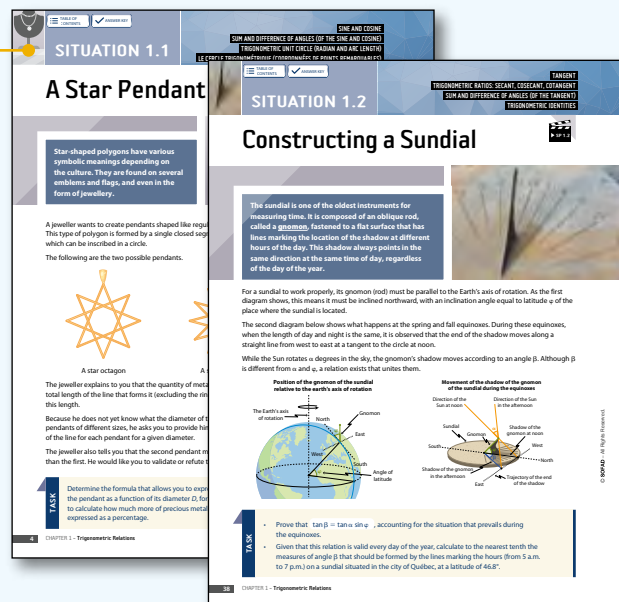
The first page describes the context and theme that will serve as a backdrop for the acquisition of the new knowledge discussed in the chapter.



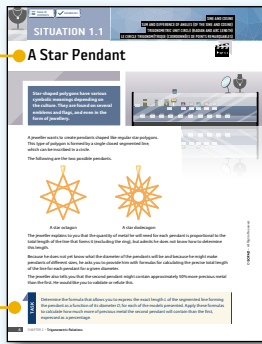
A table of contents accompanies this first page. The knowledge to be acquired is described for each of the *Situations*, as well as the theme of the situational problems.

SITUATIONS

In general, there are two learning *Situations* per chapter. The approach taken in these situations makes it possible to acquire new knowledge and develop mathematical skills in real, realistic or purely mathematical contexts.



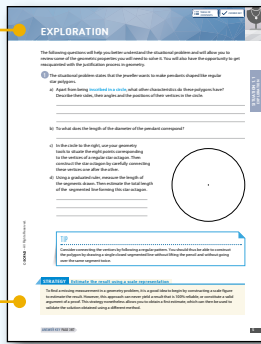
PHASES OF EACH SITUATION



SITUATIONAL PROBLEM

Linked to the main theme of the chapter, this page briefly describes the context of the situational problem, as well as the information required to solve it.

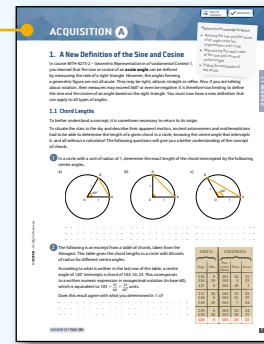
A box describes the task you will have to perform later in the *Solution* section. This task is the starting point for acquiring new knowledge to solve the situational problem.



EXPLORATION

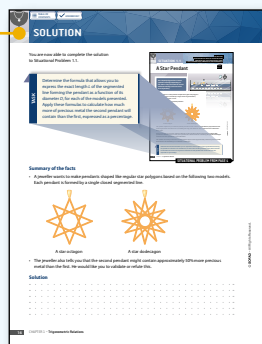
This section invites you to analyze the data of a situational problem, and then to identify the knowledge that you possess and the knowledge you need to acquire in order to perform the task.

The questions posed will guide you toward a problem-solving strategy.



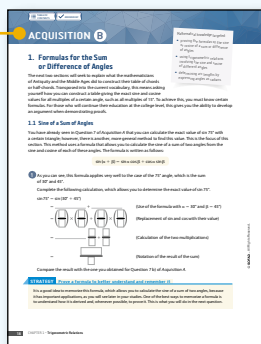
ACQUISITION A

This is where the knowledge needed to solve the situational problem is assimilated. Each *Acquisition* encourages reflection before presenting new mathematical knowledge.



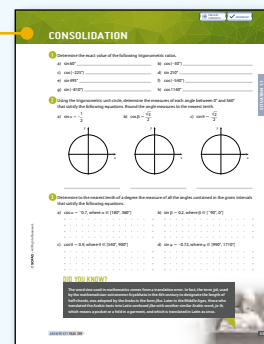
SOLUTION

By the time you reach this section, you should have acquired all the knowledge and strategies that are essential to solving the situational problem described at the beginning of the situation.



ACQUISITION B

In this second acquisition, you will acquire new knowledge prescribed by the program linked to the knowledge encountered in *Acquisition A*.



CONSOLIDATION

This section will allow you to consolidate the mathematical knowledge acquired in *Acquisitions A* and *B*. As in the *Integration* section, this *Consolidation* also contributes to the development of mathematical skills.

AT THE END OF A CHAPTER...

KNOWLEDGE SUMMARY

This section summarizes all the knowledge to *Remember* in the form of fill-in-the-blank questions. We invite you to fill in the missing information.

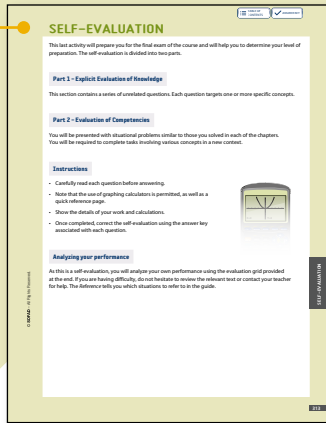
INTEGRATION

In this section, which includes exercises and complex situations, you will have to apply the knowledge seen in this chapter.

LES

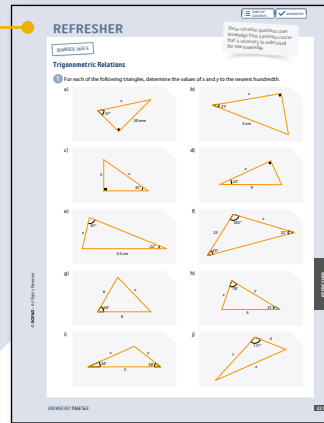
The *LES* is a complex task developed according to the certification evaluation model. It is accompanied by a competency evaluation grid.

COMPLEMENTS



SELF-EVALUATION

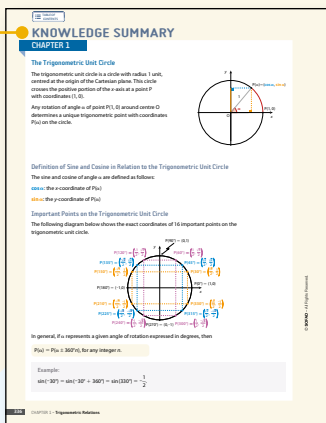
A *Self-evaluation* is presented in the first part of the *Complements* in Volume 2. It allows you to evaluate your acquired knowledge and the mathematical skills you have developed throughout the course. In this way, you will be able to identify the knowledge that you have mastered and that for which a revision is necessary before moving on to the *Summary Scored Activity*.



REFRESHER

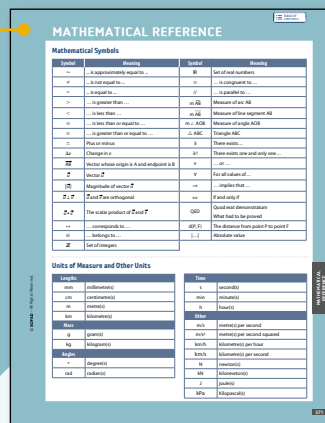
Throughout the *Situations*, you will come across headings entitled *Reminder*. These sections present concepts seen in a previous course that are necessary to understand the new knowledge or to solve the current situation.

The *Refresher* section allows you to use exercises to review the mathematical rules and concepts that are the subject of a *Reminder*.



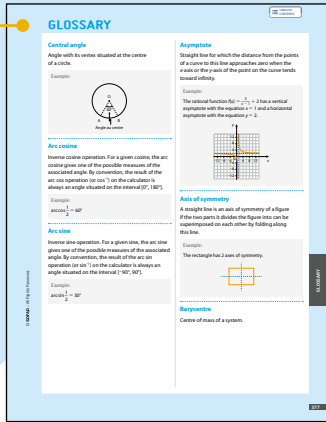
KNOWLEDGE SUMMARY

The full version of the *Knowledge Summary* is found in this section. A printable version is also available online.



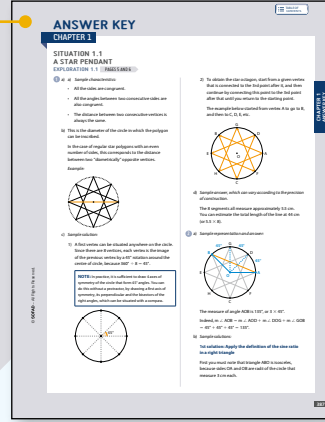
MATHEMATICAL REFERENCE

In this section, we present mathematical symbols used in the guide and some abbreviations of units of measurement. Reminders of mathematical formulas are also provided.



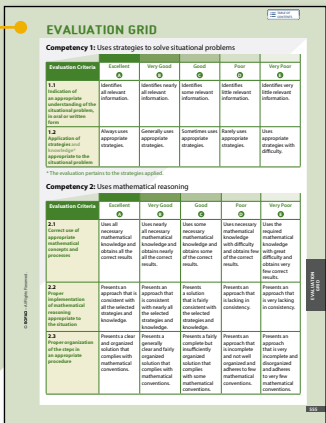
GLOSSARY

Words and expressions **written in blue** in the current text are defined in the *Glossary*.



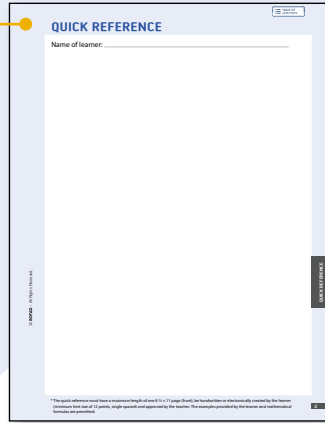
ANSWER KEY

Toward the end of the guide, you will find the *Answer Key*. It is designed not only for checking your answers, but also to complement your learning process. It contains the answers to questions and detailed explanations of the approach to be taken or the reasoning to be used.



EVALUATION GRID

A competency *Evaluation Grid* is available at the end of the guide. After solving an *LES*, you are asked to evaluate yourself using this grid. You can then complete the abbreviated version at the bottom of each *LES*.



QUICK REFERENCE

You can create your own quick reference guide. A detachable sheet is provided for this purpose at the end of the guide. You may use this quick reference during the final test.

HEADINGS AND PICTOGRAMS



Invites the student to watch a video clip on the situational problem.

TASK

Determine the formula that allows you to express the exact length $L...$

Presents the task to be performed as part of your Situational Problem.

REMINDER

REFRESHER EXERCISES
PAGE 329, QUESTIONS 6 AND 7

Solving...

The quadratic formula...

Example:

Solve the equation...

Refers to knowledge you have acquired in previous courses and refresher exercises related to this *Reminder*.

REMEMBER

Definition of the Sine...

The **trigonometric unit circle** ...

Example:

The **sine law** allows you...

Presents the mathematical knowledge you will be required to master. This is the knowledge prescribed by the study program.

STRATEGY **Estimate a...**

To determine a missing measure in a geometry problem,...

Presents problem-solving strategies that can be applied to a variety of situations.

DID YOU KNOW?

Although the number 3438 may seem strange, Aryabhata referred to it, ...

Allows you to discover historical and cultural information related to the mathematical concepts being studied.

TIP

It is important to note that this definition of the sine and the cosine in a circle of radius 1 does not contradict what you...

Provides a tip that simplifies the task, or offers a different way of dealing with the problem or of applying the concept being studied.

NOTE

This first chapter concerning the concepts involved in trigonometric relations and proofs is the most difficult...

Warns of traps to avoid or exceptions that may apply to the concept being studied.

ICT

In ICT activity 2.1.1, you can observe composites of translation sequences using GeoGebra. Find the activity on portailsofad.com...

Prompts you to complete an online activity (GeoGebra or graphing calculator) that will encourage you to explore the concept studied using technological tools.

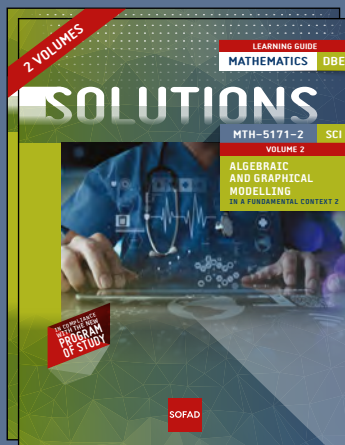
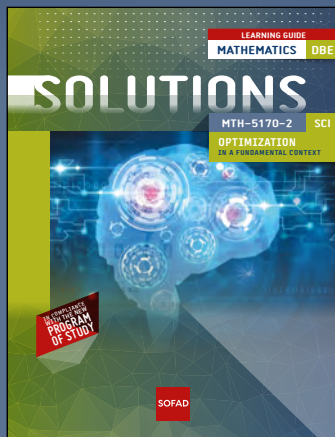
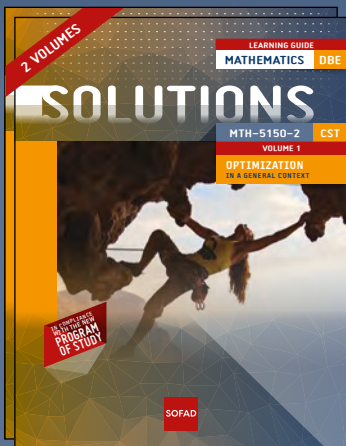
SCORED ACTIVITY

You must now complete Scored Activity 1. It can be found on the course website...

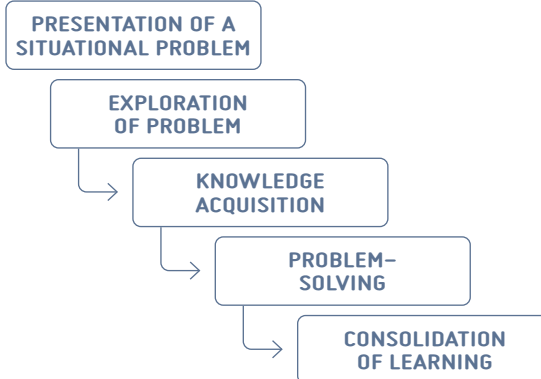
Indicates that you are ready to complete the *Scored Activity* designed to assess your comprehension as you learn. The *Summary Scored Activity* is completed at the very end of the course. These activities are presented in separate booklets of the guide. You will have to submit each completed activity to your teacher or tutor who will provide you with feedback following correction.

SOLUTIONS

The **SOLUTIONS** series covers all the courses in the Diversified Basic Education Program, including the Secondary V *Cultural, Social and Technical (CST)* and *Science (Sci)* options.



The **SOLUTIONS** learning approach is based on the acquisition of all the prescribed mathematical knowledge in a problem-solving context. The learning sequence that supports this approach is as follows:



Inductive and deductive questions give meaning to the knowledge and strategies to be acquired. The learning guides offer a multitude of simple exercises and more complex tasks to meet the needs expressed by learners and teachers. Additional resources are also available on portailsofad.com.

Components of the **SOLUTIONS** series:

- Learning guide: print and PDF versions;
- Teaching guide (PDF);
- Videos on situational problems;
- ICT activities: GeoGebra, graphing calculator;
- Scored activities;
- Answer keys.

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