

SOLUTIONS

**ALGEBRAIC
AND GRAPHICAL
MODELLING
IN A GENERAL CONTEXT 2**



SOFAD

LEARNING GUIDE

MATHEMATICS

DBE

SOLUTIONS

MTH-5151-1

CST

ALGEBRAIC
AND GRAPHICAL
MODELLING
IN A GENERAL CONTEXT 2

SOFAD

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Legend: r = right c = centre l = left
 t = top b = bottom

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HOW THE LEARNING GUIDE IS STRUCTURED

Welcome to the Learning Guide for the ***Algebraic and Graphical Modelling in a General Context 2*** course. The aim of this course, which is the second in the ***Secondary V Cultural, Social and Technical*** sequence, is to develop your ability to handle situations that require an algebraic or graphical model to express a relationship between quantities. To achieve this, you will be introduced to financial mathematics, more specifically:

- simple interest
- compound interest
- capitalization
- discounting
- interest periods

You will complete your learning by expanding your knowledge of:

- powers and logarithms
- solving exponential and logarithmic equations

You will be required to use various solution strategies to understand and model situational problems. You will need to use your mathematical reasoning skills. You will also have to describe how you solved these problems clearly and thoroughly using mathematical language.

You are now invited to complete the learning activities found in the two chapters of this guide and enrich your knowledge of algebra.

Portailsofad.com

Go to portailsofad.com for videos, ICT activities and printable versions of resources that are complementary to the SOLUTIONS series, which you can use throughout your learning journey.



CHAPTER COMPONENTS

The learning process followed in each chapter enables students to progress by building on what they have learned from one section to the next. The following diagrams illustrate this approach and specify the pedagogical intent of each section.

CHAPTER INTRODUCTION

The first page describes the context and theme that will serve as a backdrop for the acquisition of the new knowledge discussed in the chapter.



A table of contents accompanies this first page. The knowledge to be acquired is described for each of the *Situations*, as well as the theme of the situational problems.

SITUATIONS

In general, there are two learning *Situations* per chapter. The approach taken in these situations makes it possible to acquire new knowledge and develop mathematical skills in real, realistic or purely mathematical contexts.



PHASES OF EACH SITUATION



SITUATIONAL PROBLEM

Linked to the main theme of the chapter, this page briefly describes the context of the situational problem, as well as the information required to solve it.

A box describes the task you will have to perform later in the *Solution* section. This task is the starting point for acquiring new knowledge to solve the situational problem.



EXPLORATION

This section invites you to analyze the data of a situational problem, and then to identify the knowledge that you possess and the knowledge you need to acquire in order to perform the task.

The questions posed will guide you toward a problem-solving strategy.



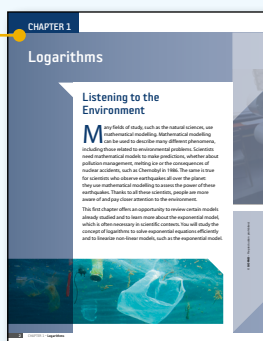
ACQUISITION A

This is where the knowledge needed to solve the situational problem is assimilated. Each *Acquisition* encourages reflection before presenting new mathematical knowledge.



SOLUTION

By the time you reach this section, you should have acquired all the knowledge and strategies that are essential to solving the situational problem described at the beginning of the situation.



ACQUISITION B

In this second acquisition, you will acquire new knowledge prescribed by the program linked to the knowledge encountered in *Acquisition A*.



CONSOLIDATION

This section will allow you to consolidate the mathematical knowledge acquired in *Acquisitions A* and *B*. As in the *Integration* section, this *Consolidation* also contributes to the development of mathematical skills.

AT THE END OF A CHAPTER...

KNOWLEDGE SUMMARY

This section summarizes all the knowledge to *Remember* in the form of fill-in-the-blank questions. We invite you to fill in the missing information.

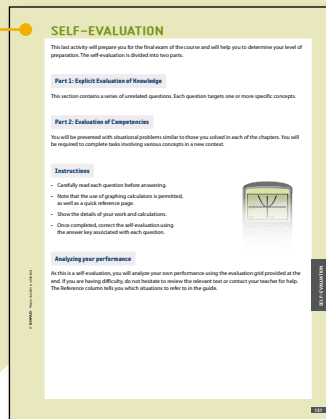
INTEGRATION

In this section, which includes exercises and complex situations, you will have to apply the knowledge seen in this chapter.

LES

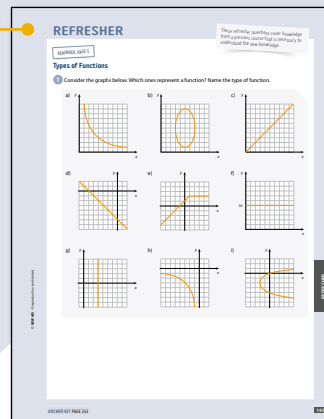
The *LES* is a complex task developed according to the certification evaluation model. It is accompanied by a competency evaluation grid.

COMPLEMENTS



SELF-EVALUATION

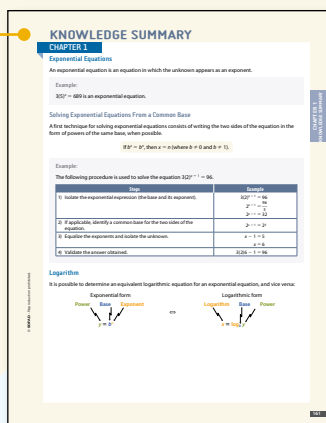
A *Self-Evaluation* section can be found in the first part of the *Complements* section. It allows you to evaluate your acquired knowledge and the mathematical skills you have developed throughout the course. In this way, you will be able to identify the knowledge that you have mastered and that for which a revision is necessary before moving on to the *Summary Scored Activity*.



REFRESHER

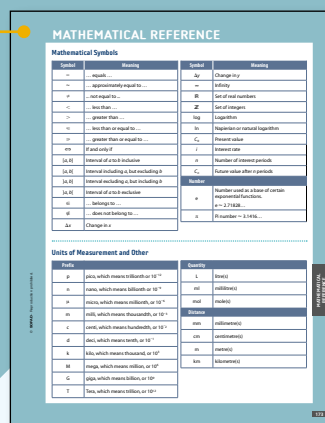
Throughout the *Situations*, you will come across headings entitled *Reminders*. These sections present concepts seen in a previous course that are necessary to understand the new knowledge or to solve the current situation.

The *Refresher* section allows you to use exercises to review the mathematical rules and concepts that are the subject of a *Reminder*.



KNOWLEDGE SUMMARY

The full version of the *Knowledge Summary* is found in this section. A printable version is also available online.



MATHEMATICAL REFERENCE

In this section, we present mathematical symbols used in the guide and some abbreviations of units of measurement. Reminders of mathematical formulas are also provided.

HEADINGS AND PICTOGRAMS



Invites the student to watch a video clip on the situational problem.

TASK

You must determine at what precise times of the day...

Presents the task to be performed as part of your Situational Problem.

REMINDER

REFRESHER EXERCISES
PAGES 145 TO 151, QUESTIONS 1 TO 22

Types of Functions

The following are some functional models studied in previous courses.

Refers to knowledge you have acquired in previous courses and refresher exercises related to this *Reminder*.

REMEMBER

Exponential Equations

An exponential equation is an equation in which the unknown appears as an exponent.

Presents the mathematical knowledge you will be required to master. This is the knowledge prescribed by the study program.

STRATEGY Represent...

When a table of values relates two variables and you are looking for...

Presents problem-solving strategies that can be applied to a variety of situations.

DID YOU KNOW?

The word *logarithm* is composed of two words of Greek origin. *Logos*, which means...

Allows you to discover historical and cultural information related to the mathematical concepts being studied.

TIP

Remember that in the rule of an exponential function written as $f(x) = ab^x$, parameter a represents the initial value...

Provides a tip that simplifies the task, or offers a different way of dealing with the problem or of applying the concept being studied.

CAUTION!

In this type of expression, you must take into account the order of operations. For example, $5(2)^x \neq 10^x$, of the same...

Warns of traps to avoid or exceptions that may apply to the concept being studied.

ICT

ICT Activity 1.1.1 allows you to explore the keys that are used to manipulate exponential and logarithmic expressions on a graphing calculator.

Prompts you to complete an online activity (GeoGebra or graphing calculator) that will encourage you to explore the concept studied using technological tools.

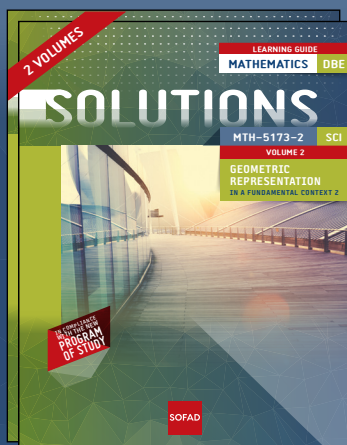
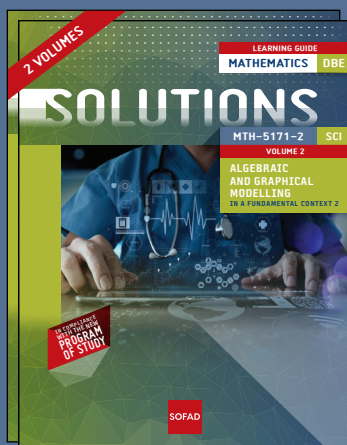
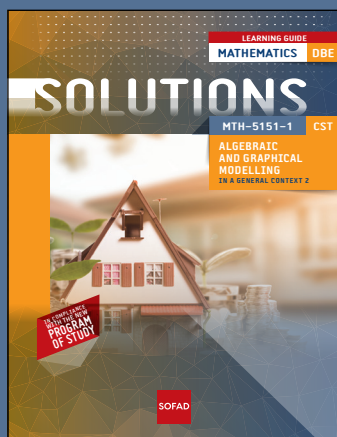
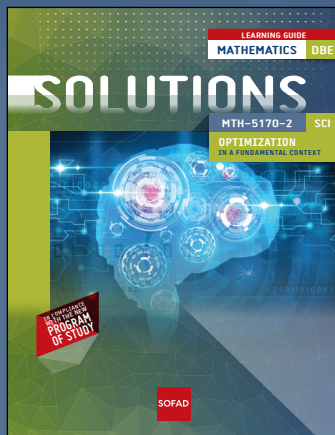
SCORED ACTIVITY

You must now complete Scored Activity 1 on Chapter 1. Find this activity on ...

Indicates that you are ready to complete the *Scored Activity* designed to assess your comprehension as you learn. The *Summary Scored Activity* is completed at the very end of the course. These activities are presented in separate booklets of the guide. You will have to submit each completed activity to your teacher or tutor who will provide you with feedback following correction.

SOLUTIONS

The **SOLUTIONS** series covers all the courses in the Diversified Basic Education Program, including the Secondary V Cultural, Social and Technical (CST) and Science (Sci) options.



The **SOLUTIONS** learning approach is based on the acquisition of all the prescribed mathematical knowledge in a problem-solving context. The learning sequence that supports this approach is as follows:

PRESENTATION OF A SITUATIONAL PROBLEM

EXPLORATION OF PROBLEM

KNOWLEDGE ACQUISITION

PROBLEM-SOLVING

CONSOLIDATION OF LEARNING

Inductive and deductive questions give meaning to the knowledge and strategies to be acquired. The learning guides offer a multitude of simple exercises and more complex tasks to meet the needs expressed by learners and teachers. Additional resources are also available on portailsofad.com.

Components of the **SOLUTIONS** series:

- Learning guide: print and PDF versions;
- Teaching guide (PDF);
- Videos on situational problems;
- ICT activities: GeoGebra, graphing calculator;
- Scored activities;
- Answer keys.