SOLUTIONS

ALGEBRAIC AND GRAPHICAL MODELLING IN A GENERAL CONTEXT 2

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ALGEBRAIC AND GRAPHICAL MODELLING IN A GENERAL CONTEXT 2



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Legend: r = right c = centre l = leftt = top b = bottom

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HOW THE LEARNING GUIDE IS STRUCTURED

Welcome to the Learning Guide for the *Algebraic and Graphical Modelling in a General Context 2* course. The aim of this course, which is the second in the **Secondary V** *Cultural, Social and Technical* sequence, is to develop your ability to handle situations that require an algebraic or graphical model to express a relationship between quantities. To achieve this, you will be introduced to financial mathematics, more specifically:

- simple interest
- compound interest
- capitalization
- discounting
- interest periods

You will complete your learning by expanding your knowledge of:

- powers and logarithms
- · solving exponential and logarithmic equations

You will be required to use various solution strategies to understand and model situational problems. You will need to use your mathematical reasoning skills. You will also have to describe how you solved these problems clearly and thoroughly using mathematical language.

You are now invited to complete the learning activities found in the two chapters of this guide and enrich your knowledge of algebra.

Portailsofad.com

Go to <u>portailsofad.com</u> for videos, ICT activities and printable versions of resources that are complementary to the SOLUTIONS series, which you can use throughout your learning journey.



CHAPTER COMPONENTS

The learning process followed in each chapter enables students to progress by building on what they have learned from one section to the next. The following diagrams illustrate this approach and specify the pedagogical intent of each section.

CHAPTER INTRODUCTION

The first page describes the context and theme that will serve as a backdrop for the acquisition of the new knowledge discussed in the chapter.



A table of contents accompanies this first page. The knowledge to be acquired is described for each of the *Situations*, as well as the theme of the situational problems.



SITUATIONS

In general, there are two learning *Situations* per chapter. The approach taken in these situations makes it possible to acquire new knowledge and develop mathematical skills in real, realistic or purely mathematical contexts.

VI

PHASES OF EACH SITUATION



SITUATIONAL PROBLEM

Linked to the main theme of the chapter, this page briefly describes the context of the situational problem, as well as the information required to solve it.

 A box describes the task you will have to perform later in the *Solution* section.
 This task is the starting point for acquiring new knowledge to solve the situational problem.



- EXPLORATION

This section invites you to analyze the data of a situational problem, and then to identify the knowledge that you possess and the knowledge you need to acquire in order to perform the task.

 The questions posed will guide you toward a problem-solving strategy.



ACQUISITION A

This is where the knowledge needed to solve the situational problem is assimilated. Each *Acquisition* encourages reflection before presenting new mathematical knowledge.



- SOLUTION

By the time you reach this section, you should have acquired all the knowledge and strategies that are essential to solving the situational problem described at the beginning of the situation.



- ACQUISITION B

In this second acquisition, you will acquire new knowledge prescribed by the program linked to the knowledge encountered in *Acquisition A*.

_	CHAPTER 1		
	Logarit	hms	-
		Listening to the Environment	
		The first order of the subsystem of the	

CONSOLIDATION

This section will allow you to consolidate the mathematical knowledge acquired in *Acquisitions A* and *B*. As in the *Integration* section, this *Consolidation* also contributes to the development of mathematical skills.

AT THE END OF A CHAPTER...

KNOWLEDGE SUMMARY

This section summarizes all the knowledge to *Remember* in the form of fill-in-the-blank questions. We invite you to fill in the missing information.

INTEGRATION

In this section, which includes exercises and complex situations, you will have to apply the knowledge seen in this chapter.

LES

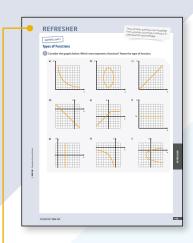
The *LES* is a complex task developed according to the certification evaluation model. It is accompanied by a competency evaluation grid.

COMPLEMENTS



SELF-EVALUATION

A *Self-Evaluation* section can be found in the first part of the *Complements* section. It allows you to evaluate your acquired knowledge and the mathematical skills you have developed throughout the course. In this way, you will be able to identify the knowledge that you have mastered and that for which a revision is necessary before moving on to the *Summary Scored Activity*.



REFRESHER

Throughout the *Situations*, you will come across headings entitled *Reminders*. These sections present concepts seen in a previous course that are necessary to understand the new knowledge or to solve the current situation.

The *Refresher* section allows you to use exercises to review the mathematical rules and concepts that are the subject of a *Reminder*.

	CHAPTER 1 Exponential Equations	
	An exponential equation is an equation in which the unknown appear	as an exponent.
	Example:	
	$3 \langle S \rangle^{\mu} = 689$ is an exponential equation.	
	Solving Exponential Equations From a Common Base	
	A first technique for solving exponential equations consists of writing to form of powers of the same base, when possible.	he two sides of the equation in the
	If $b^{\mu}=b^{\mu},$ then $x=n$ (where $b\neq 0$ and	& ≠ 1).
	Example:	
	. The following procedure is used to solve the equation $3 2 ^{\nu-1}=96$	
	Steps	Example
	 solate the exponential expression (the base and its exponent). 	$2(2)^{r+1} = 66$ $2^{r+1} = \frac{96}{1}$ $2^{r+1} = 12$
	 If applicable, identify a common base for the two sides of the equation. 	2 = 24
	 Equalize the exponents and isolate the unknown. 	x-1=5 x=6
	4 Validate the answer obtained.	3(2)6 - 1 = 96
0 00440 - Pay maked as principles	Legarithm Raposate termine an explosited toget thric expectator for an ex- Equatorial form Party and the constant ex- y = 1	ponential equation, and vice versa: Logarithmic form with the power a = log_y

KNOWLEDGE SUMMARY

The full version of the *Knowledge Summary* is found in this section. A printable version is also available online.

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> greater than log Logarithm
« less than or equal to In Napierian or natural logalithm
Image: Second
en If and only if i interest rate
(a, b) Interval of a to b inclusive n Number of interest periods
[4, b] Interval including a, but excluding b C, Future value after n periods
[a, b] Interval excluding a, but including b Number
ja, b(interval of a to b-soclusive Number used as a base of certain exponential functions.
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iii does not belong to a Rinumber ~ 2.1416
Δx Change in x
Units of Measurement and Other Potis Coastly Coastly
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P micro, which means millionth, or 10 ⁻⁴ mol mole(s)
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MATHEMATICAL REFERENCE

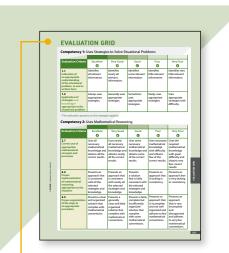
In this section, we present mathematical symbols used in the guide and some abbreviations of units of measurement. Reminders of mathematical formulas are also provided.

Words and expressions written in blue in the current text are defined in the *Glossary*.

EMAPTER 1 SITUATION 1.1 A DECONTAMINATION OPERATION EXPLORATION 1.1 MAKESTRE		
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ANSWER KEY

Toward the end of the guide, you will find the Answer Key. It is designed not only for checking your answers, but also to complement your learning process. It contains the answers to questions and detailed explanations of the approach to be taken or the reasoning to be used.



EVALUATION GRID

A competency *Evaluation Grid* is available at the end of the guide. After solving an *LES*, you are asked to evaluate yourself using this grid. You can then complete the abbreviated version at the bottom of each *LES*.

	Name of learner:	
	Parties of Parties.	
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ł		30
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QUICK REFERENCE

You can create your own quick reference guide. A detachable sheet is provided for this purpose at the end of the guide. You may use this quick reference during the final test.

HEADINGS AND PICTOGRAMS



TASK

Invites the student to watch a video clip on the situational problem.

You must determine at what precise times of the day...

Presents the task to be performed as part of your Situational Problem.

REMINDER

PAGES 145 TO 151, QUESTIONS 1 TO 22

REFRESHER EXERCISES

Types of Functions

The following are some functional models studied in previous courses.

Refers to knowledge you have acquired in previous courses and refresher exercises related to this *Reminder*.

REMEMBER

Exponential Equations

An exponential equation is an equation in which the unknown appears as an exponent.

Presents the mathematical knowledge you will be required to master. This is the knowledge prescribed by the study program.

STRATEGY Represent...

When a table of values relates two variables and you are looking for...

Presents problem-solving strategies that can be applied to a variety of situations.

The word *logarithm* is composed of two words of Greek origin. *Logos*, which means...

Allows you to discover historical and cultural information related to the mathematical concepts being studied.

TIP

Remember that in the rule of an exponential function written as $f(x) = ab^x$, parameter *a* represents the initial value...

Provides a tip that simplifies the task, or offers a different way of dealing with the problem or of applying the concept being studied.

CAUTION!

In this type of expression, you must take into account the order of operations. For example, $5(2)^x \neq 10^x$, of the same...

Warns of traps to avoid or exceptions that may apply to the concept being studied.

ICT

ICT Activity 1.1.1 allows you to explore the keys that are used to manipulate exponential and logarithmic expressions on a graphing calculator.

Prompts you to complete an online activity (GeoGebra or graphing calculator) that will encourage you to explore the concept studied using technological tools.

SCORED

You must now complete Scored Activity 1 on Chapter 1. Find this activity on ... Indicates that you are ready to complete the Scored Activity designed to assess your comprehension as you learn. The Summary Scored Activity is completed at the very end of the course. These activities are presented in separate booklets of the guide. You will have to submit each completed activity to your teacher or tutor who will provide you with feedback following correction.

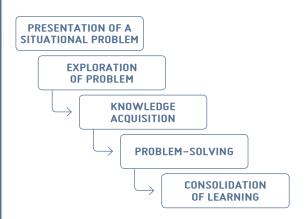
SOLUTIONS

The **SOLUTIONS** series covers all the courses in the Diversified Basic Education Program, including the Secondary V *Cultural, Social and Technical* (CST) and *Science* (Sci) options.



SOFAD

The **SOLUTIONS** learning approach is based on the acquisition of all the prescribed mathematical knowledge in a problem-solving context. The learning sequence that supports this approach is as follows:



Inductive and deductive questions give meaning to the knowledge and strategies to be acquired. The learning guides offer a multitude of simple exercises and more complex tasks to meet the needs expressed by learners and teachers. Additional resources are also available on <u>portailsofad.com</u>.

Components of the SOLUTIONS series:

- · Learning guide: print and PDF versions;
- Teaching guide (PDF);
- · Videos on situational problems;
- ICT activities: GeoGebra, graphing calculator;
- Scored activities;
- Answer keys.