

IN A GENERAL CONTEXT



SOFAD



IN A GENERAL CONTEXT



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Legend: r = right c = centre I = leftt = top b = bottom

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# HOW THE LEARNING GUIDE IS STRUCTURED

Welcome to the learning guide for the *Optimization in a General Context* course. The aim of this course, which is the first in the **Secondary V Cultural, Social and Technical** sequence, is to develop your skills in dealing with situations that require optimal solutions. To achieve this, you will study linear programming, namely:

- systems of first-degree inequalities in two variables
- the representation of constraints
- the feasible region
- the objective or economic function

You will continue your learning by expanding your knowledge of:

- the cosine law
- equivalent figures

You will then discover new knowledge about graphs.

You will be required to use various solution strategies to understand and model situational problems. You will need to use your mathematical reasoning skills. You will also have to describe how you solved these problems clearly and thoroughly using mathematical language.

You are now invited to complete the learning activities found in the two chapters of Volume 1 of this guide and enrich your knowledge of optimization.

#### Portailsofad.com

Go to portailsofad.com for videos, ICT activities and printable versions of resources that are complementary to the SOLUTIONS series, which you can use throughout your learning journey.



# **CHAPTER COMPONENTS**

The learning process followed in each chapter enables students to progress by building on what they have learned from one section to the next. The following diagrams illustrate this approach and specify the pedagogical intent of each section.

#### **CHAPTER INTRODUCTION**

The first page describes the context and theme that will serve as a backdrop for the acquisition of the new knowledge discussed in the chapter.



A table of contents accompanies this first page. The knowledge to be acquired is described for each of the *Situations*, as well as the theme of the situational problems.

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SITUATION 1.1

#### SITUATIONS

In general, there are two learning *Situations* per chapter. The approach taken in these situations makes it possible to acquire new knowledge and develop mathematical skills in real, realistic or purely mathematical contexts.

VI

# PHASES OF EACH SITUATION



#### SITUATIONAL PROBLEM

Linked to the main theme of the chapter, this page briefly describes the context of the situational problem, as well as the information required to solve it.

A box describes the task you will have to perform later in the *Solution* section. This task is the starting point for acquiring new knowledge to solve the situational problem.

The questions is the explosition activity will help you to analyze the situational peddem and leaps solving it. They will also also you to even certain concepts scale is the definition of variables and equations, is well
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whether interface practices are compared in this case are some or at at the accurate physical method, and include the physical method and an anti-physical method. The second and a second

#### **EXPLORATION**

This section invites you to analyze the data of a situational problem, and then to identify the knowledge that you possess and the knowledge you need to acquire in order to perform the task.

The questions posed will guide you toward a problem-solving strategy.

1. I Not he index	Expressing a Situation as an Inequality or already learned has to repare a statistic as a first degree reparties analysis for bilinging questions of the hyposite dendrings prove shifts reading strations as first-degree inequalities in fore strategies.	<ul> <li>representation and a statement of an a livel-dependence comparing = line vertains,         = "Proceeding the solution of of a livel-dependence of the solution of of a livel-dependence on the solution.     </li> </ul>
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#### **ACQUISITION A**

This is where the knowledge needed to solve the situational problem is assimilated. Each *Acquisition* encourages reflection before presenting new mathematical knowledge.

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Sar										1254								 				
San										1.254												
San										1.15												
5a										120												

#### SOLUTION

By the time you reach this section, you should have acquired all the knowledge and strategies that are essential to solving the situational problem described at the beginning of the situation.

$\label{eq:constraint} \begin{array}{c} \textbf{1. The Case of Discrete Variables} \\ The black from densities taking in which and the constraints of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the statistic of the product of the black from the product of the black from the product of the black from the black from the product of the black from the product of the black from the black from the product of the black from the product of the black from $	<ul> <li>exploring by designer symplectic products and descripting and descripting and descripting and and any product of end product and program of end products and end products and end products and end products and end products and end products and end products and any products and end products and end products and end products and end products and end products and end products and end products and end products and end products and end pr</li></ul>	
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#### **ACQUISITION B**

In this second acquisition, you will acquire new knowledge prescribed by the program linked to the knowledge encountered in *Acquisition A*.

<ul> <li>a) Nucle warks to boy a maximum of tester</li> </ul>	with each situation, having first do • the number of blauses as parts.	fixed the satisfies.
	1) $x \gg 2y$	2) x <1 2y
*	1) x < 2y	4) x>2y
<li>b) The lengths of the scales of a triangle will measures if on.</li>	haperineter of at least 13 cm, ar	d where one cide
*	1) x+y+x> m	2y = x + y + x < xx
7	1) x+y+8<10	4 x+y+3>33
c) The revenue from the talk of lattery talk cold a certain number of lattery talkets a callecase at least \$920.	ets by a convenience store clerk. I d \$2 per unit, and a certain numb	n Univer hasars, the slenk er al \$3 per unit. His tatal
	$1)  2w + 1y \gg 400$	2y = 2x + 5y < 400
7	31 2x + 3y > 400	4) 2x + 5y < 400
for each of the following ciptements.		
1) Define the variables being related.		
2) Identify and highlight an expression I	that allows you to determine the	inequality sign.
2) Express the statement as an inequality	lyin hei sielakles.	
<ul> <li>A lady which knitting uses two types of the must use at least three times more at</li> </ul>	pare to level a scarf. To create the s I pare A than of pare 3.	natif the wards,
1 e		
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2) Highlight		1
<ol> <li>trepality</li> </ol>		
b) Availa has been jubs. The generally specific forward of advances as a work with a series.	dc a maximum of five hours per v	eed mane aca d
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T Multi-Mult		
Ti basedite		

#### **CONSOLIDATION**

This section will allow you to consolidate the mathematical knowledge acquired in *Acquisitions A* and *B*. Like the *Integration* section, *Consolidation* also helps with the development of mathematical skills.

#### AT THE END OF A CHAPTER...

#### **KNOWLEDGE SUMMARY**

This section summarizes all the knowledge to *Remember* in the form of fill-in-the-blank questions. We invite you to fill in the missing information.

#### INTEGRATION

In this section, which includes exercises and complex situations, you will have to apply the knowledge seen in this chapter.

#### LES

The *LES* is a complex task developed according to the certification evaluation model. It is accompanied by a competency evaluation grid.

# COMPLEMENTS



#### SELF-EVALUATION

A *Self-Evaluation* section can be found in the first part of the *Complements* in Volume 2. It allows you to evaluate your acquired knowledge and the mathematical skills you have developed throughout the course. In this way, you will be able to identify the knowledge that you have mastered and that for which a revision is necessary before moving on to the *Summary Scored Activity*.

•	REFRESHER		These retruction quantizers report knowledge from Administrationary basis and the sectors	
	REMEMPER, PAGE 6		Maintailedge.	
	Graphing a Line Defined by	an Equation		
	Represent each of the following	g rules in a graph using the interce	ots and another point as required.	
	a) $4x + 3y = 24$	b) $5x - 2y = 0$	c) $y = \frac{3}{2}x - 4$	
	d  −7x + 7y − 21 = 0	a) y = -3.5	f) <sup>2</sup> / <sub>2</sub> = 3	
	Graph each of the following rule	les using the values of parameters	e and b:	
	<li>a) y = 2x</li>	b) $y = -\frac{1}{2}x$	- 2	
	c) $y = \frac{3x}{2} + 4$	d) y = -x -	2	
	Julian has managed to save \$1 of \$100.00 per day.	500.00 for a trip to Florida, during v	hich he expects to spend an average	
	Graph the relation between th	e amount of money remaining and	the number of days elapsed.	
	Represent the following relation	on in a graph:		
	The perimeter of a square can the quadrilateral and r is the k	be calculated using the rule $P = 4s$ ength of one of its sides.	where Prepresents the perimeter of	
	Alex is a high school physical of education. This year, because of education.	education teacher. His salary is base se Alex got a full-time teaching cont	d on his teaching experience and level ract, his salary will be \$42 000.00,	
	regardless of how many days	te works per week.		
	Represent the relation between a graph.	n the number of days worked in on	e complete year and Alex's salary in	
	A company would like to laun directors of the company wan	ch a certain product on the market. It to represent the supply and dema	After consulting a marketing firm, the of graphically. It was explained to	
	them that the more expensive a product, the less the demand from consumers. On the other hand, if			
į.	enough profit for the company	ce, it is sets appealing to put it on th y. The equilibrium point must there	e manuel because it will not generate one be determined. The marketing	
1	firm also establishes the follow	sing rules, explaining that the sellin	price of a product is considered a	
1 (TRADE 0	P. = 1400 - 3a, where P. is the	selling price in relation to demand	S) and o is the quantity of products.	
	$P_i = 2q + 150$ , where $P_i$ is the	willing price in relation to supply (\$)	nd q is the quantity of products.	
	Graph the situation and find ti	he equilibrium point between supp	y and demand.	
	ANTINGS KEY PAGE 212			

#### REFRESHER

Throughout the *Situations*, you will come across headings entitled *Reminders*. These sections present concepts seen in a previous course that are necessary to understand the new knowledge or to solve the current situation.

The *Refresher* section allows you to use exercises to review the mathematical rules and concepts that are the subject of a *Reminder*.



#### KNOWLEDGE SUMMARY

The full version of the *Knowledge Summary* is found in this section. A printable version is also available online.

 Sumbol	Newton	Sumbul	Baselon	
-	equals	d	Danuter	
~	approximately equal to	A	Area	
*	not equal to	c	Circumference	
z	Plus or minus	v	Volume	
<	less than	h	Alibutude (or Height)	
>	greater than	0	Apothem	
4	less than or equal to (maximum of, not more than, at most, etc.)	x	2.14159265258879	
	greater than or equal to (minimum of, not less than, at least, etc.)	d(A,B)	in a graph, distance between a vertex A and a vertex B	
-/2	Square root	A-2	in a graph, path formed by the edge that connects vertices A and B	
4	belongs to	_ ABC	Triangle ABC	
z	Set of integers	2.A	Angle A	
п	Set of real numbers	n Ali	Length of segment AB	
N	Set of natural numbers	-	Periodic Example 1.6 = 1.6666 1.254 = 1.234234234	
10	Slope	1.1	Degree	
/	Radius			·
				to broad estimate site Oracle 0

#### MATHEMATICAL REFERENCE

In this section, we present mathematical symbols used in the guide and some abbreviations of units of measurement. Reminders of mathematical formulas are also provided.



Words and expressions written in blue in the current text are defined in the *Glossary*.

CHAPTER 1 SITUATION 1.1 — THE COMMUNITY GARDEN		
Construction of a constru	<ul> <li>Anatomic in the state of the st</li></ul>	
ACCOUNTS 1.1.8. UNKENDED Hangto some Hangto some and an expedient and anyon taken the method and anyon anyon and anyon Markana anyon.	Departments     1    2    -References     2    2    -References     2    2    -References     2    2    2    2    2    2    2	0 KOND Paymenting periodation

#### **ANSWER KEY**

Toward the end of the guide, you will find the *Answer Key*. It is designed not only for checking your answers, but also to complement your learning process. It contains the answers to questions and detailed explanations of the approach to be taken or the reasoning to be used.



#### **EVALUATION GRID**

A competency *Evaluation Grid* is available at the end of the guide. After solving an *LES*, you are asked to evaluate yourself using this grid. You can then complete the abbreviated version at the bottom of each *LES*.

None of lamor	1	-	QUICK REFERENCE	
			Name d'Isanne	
		0 60940 New Academic Sciences		DAKK REFERENCE

#### **QUICK REFERENCE**

You can create your own quick reference guide. A detachable sheet is provided for this purpose at the end of the guide in Volume 2. You may use this quick reference during the final test.

# **HEADINGS AND PICTOGRAMS**



**IASK** 

Invites the student to watch a video clip on the situational problem.

To help the group, you must propose a ...

Presents the task to be performed as part of your Situational Problem.

REMINDER

**REFRESHER EXERCISES** PAGE 145, QUESTIONS 1 TO 6

Graphing a line...

To graph a line, you only need...

Example:

Below is how to draw...

Refers to knowledge you have acquired in previous courses and refresher exercises related to this *Reminder*.

#### REMEMBER



When you express a statement...

Example:

1. Even if you **double** the...

Presents the mathematical knowledge you will be required to master. This is the knowledge prescribed by the study program.

#### STRATEGY Representing a situation ...

To properly analyze a situation, representing it using one or more modes ...

Presents problem-solving strategies that can be applied to a variety of situations.

All commercial airliners operate in automatic steering mode ...

Allows you to discover historical and cultural information related to the mathematical concepts being studied.

#### TIP

To turn a statement into a mathematical expression, you may need to use mathematical symbols associated with the four basic operations... Provides a tip that simplifies the task, or offers a different way of dealing with the problem or of applying the concept being studied.

### CAUTION!

To ensure an accurate interpretation of a statement, units of measurement must always be specified when defining the variables...

Warns of traps to avoid or exceptions that may apply to the concept being studied.

#### ICT

ICT Activity 1.3.1 shows you how to use the **zoom** key on the graphing calculator to find replacement points if necessary ...

Prompts you to complete an online activity (GeoGebra or graphing calculator) that will encourage you to explore the concept studied using technological tools.

# SCORED ACTIVITY

You must now complete Scored Activity 1 on Chapters 1 and 2. Find this activity at... Indicates that you are ready to complete the Scored Activity designed to assess your comprehension as you learn. The Summary Scored Activity is completed at the very end of the course. These activities are presented in separate booklets of the guide. You will have to submit each completed activity to your teacher or tutor who will provide you with feedback following correction.

# SOLUTIONS

The **SOLUTIONS** series covers all the courses in the Diversified Basic Education Program, including the Secondary V *Cultural, Social and Technical* (CST) and *Science* (Sci) options.



The **SOLUTIONS** learning approach is based on the acquisition of all the prescribed mathematical knowledge in a problem-solving context. The learning sequence that supports this approach is as follows:



Inductive and deductive questions give meaning to the knowledge and strategies to be acquired. The learning guides offer a multitude of simple exercises and more complex tasks to meet the needs expressed by learners and teachers. Additional resources are also available on <u>portailsofad.com</u>.

#### Components of the SOLUTIONS series:

- · Learning guide: print and PDF versions;
- Teaching guide (PDF);
- Videos on situational problems;
- · ICT activities: GeoGebra, graphing calculator;
- Scored activities;
- Answer keys.

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