SUMMARY TEACHING GUIDE

MATHEMATICS DBE

SOLUTIONS

Secondary IV *Cultural, Social and Technical* Option

MTH-4151-1 CST

ALGEBRAIC AND GRAPHICAL MODELLING
IN A GENERAL CONTEXT 1

MTH-4152-1 CST

DATA COLLECTION
IN A GENERAL CONTEXT

MTH-4153-2 CST

GEOMETRIC REPRESENTATION
IN A GENERAL CONTEXT

IN COMPLIANCE WITH THE NEW PROGRAM OF STUDY

SOFAD

SUMMARY TEACHING GUIDE

MATHEMATICS DBE

SOLUTIONS

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SOFAD

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SOLUTIONS

INTRODUCTION

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Introduction

The **SOLUTIONS** series is designed to meet the requirements of the ***Mathematics Program of Study, Adult General Education;*** Diversified Basic Education (DBE), MEES 2017. The learning approach is focused on developing mathematical competencies by solving rich and meaningful situational problems. Learners are guided in their quest to find solutions to complex problems through the use of questions designed to help them acquire the mathematical knowledge they need.

This teaching guide provides further detail on the components of the **SOLUTIONS** learning guides.

It also offers a reminder of certain requirements of the program:

- Nature of learning activities
- Favoured teaching approach
- Families of learning situations
- Program options
- The three subject-specific competencies
- Prescribed knowledge
- Evaluation criteria targeted in the DBE course.

Lastly, the summary teaching guide contains references and notes for teachers that are relevant to each Secondary IV course.

In summary, this information is provided to guide teachers in preparing and delivering support activities to learners.

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Portailsofad.com

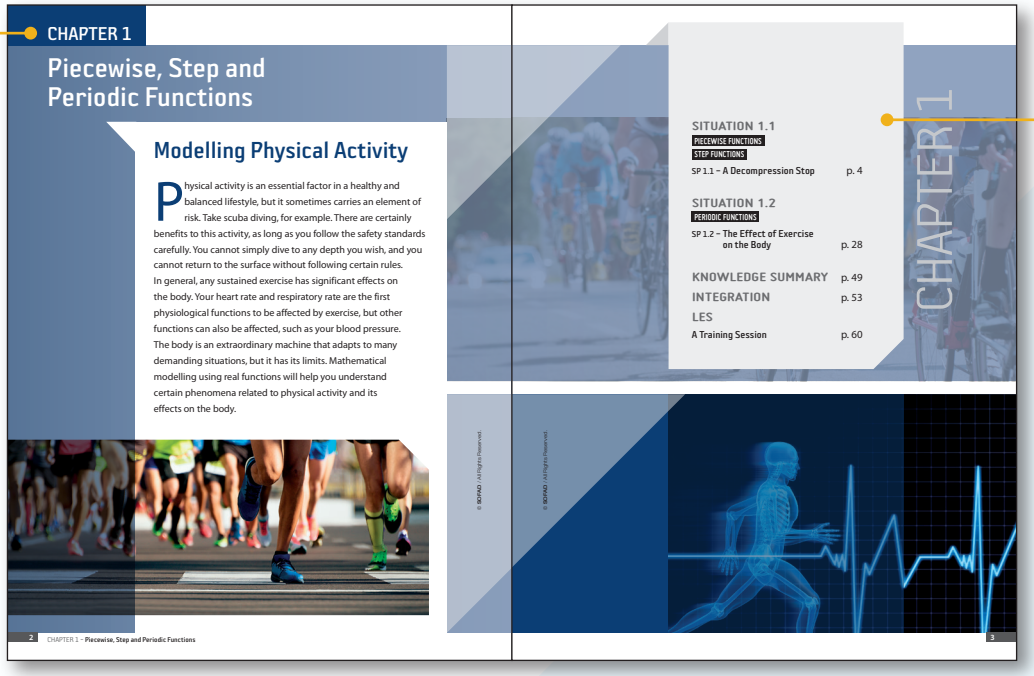
Teachers can find all the material they need to accompany the **SOLUTIONS** series on **portailsofad.com**: the digital version of the summary teaching guide, videos, ICT activities, printable versions of complementary resources, answer keys to scored activities and tracking tools.

Chapter Components

The learning process followed in each chapter of the **SOLUTIONS** series for Secondary IV is illustrated below. The pedagogical intent is specified for each section. Learners progress by building on what they have learned from one section to the next.

CHAPTER INTRODUCTION

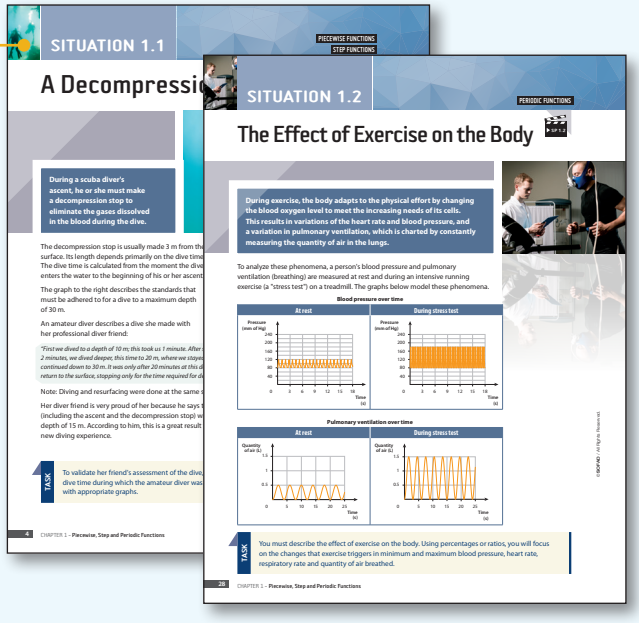
The first page describes the context and theme that will serve as a backdrop for the acquisition of the new knowledge discussed in the chapter.



A table of contents accompanies this first page. The knowledge to be acquired is described for each of the *Situations*, as well as the theme of the situational problems.

SITUATIONS

In general, each chapter contains two learning *Situations*. The approach taken in these situations enables learners to acquire new knowledge and develop mathematical skills in real, realistic or purely mathematical contexts.



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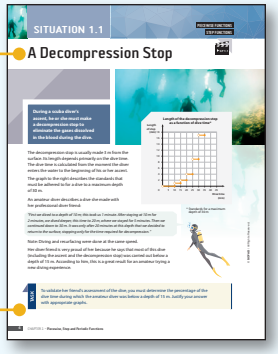
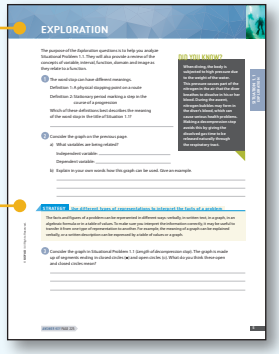
Phases of Each Situation



SITUATIONAL PROBLEM

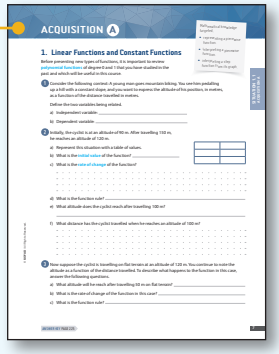
A box describes the task that the learner must perform later in the *Solution* section. This task is the starting point for acquiring new knowledge to solve the situational problem. A video is provided to accompany the situational problem.



EXPLORATION

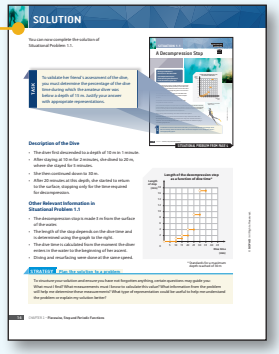
This section invites the learner to analyze the data of a situational problem, and then to identify the knowledge they possess and the knowledge they need to acquire in order to perform the task.

The questions posed will guide them toward a problem-solving strategy.



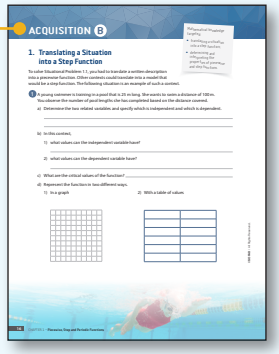
ACQUISITION A

This is where the knowledge needed to solve the situational problem is assimilated. Each *Acquisition* encourages reflection before presenting new mathematical knowledge.



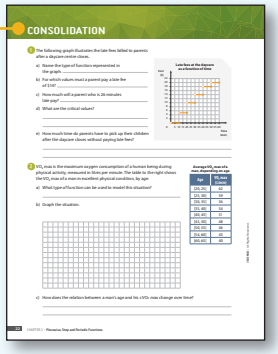
SOLUTION

When they reach this section, the learner should have acquired all the knowledge and strategies that are essential to solving the situational problem described at the beginning of the situation.



ACQUISITION B

In this second acquisition, the learner will acquire new knowledge prescribed by the program linked to the knowledge encountered in *Acquisition A*, but not required to solve the initial situational problem.



CONSOLIDATION

This section allows the learner to consolidate the mathematical knowledge acquired in *Acquisitions A* and *B*. This *Consolidation* also contributes to the development of mathematical skills.

At the End of a Chapter...

KNOWLEDGE SUMMARY

This section summarizes all the knowledge to *Remember* in the form of fill-in-the-blank questions. The learner is invited to fill in the missing information.

INTEGRATION

In this section, which includes exercises and complex situations, the learner is expected to apply the knowledge seen in this chapter. This *Integration* also contributes to the development of mathematical skills.

LES

The *LES* is a complex task developed according to the certification evaluation model. It is accompanied by a competency evaluation grid, found at the end of the learning guide, which the learner may consult. The list of observable factors is available in the course's summary teaching guide and on portailsofad.com.

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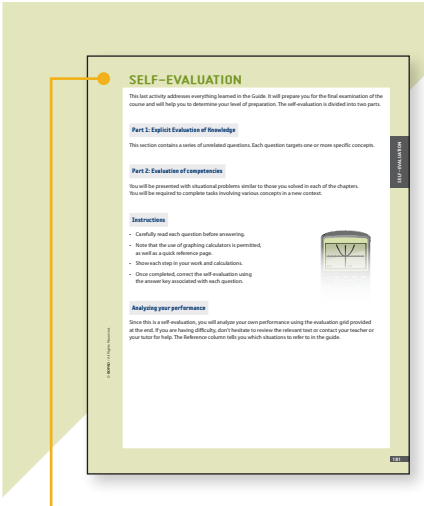
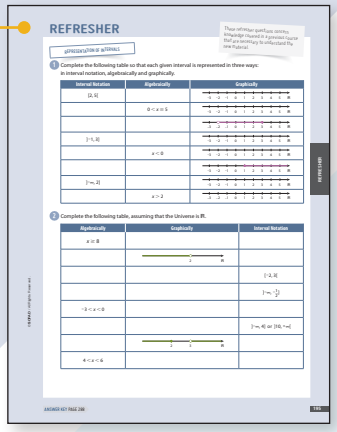
Complements



SELF–EVALUATION

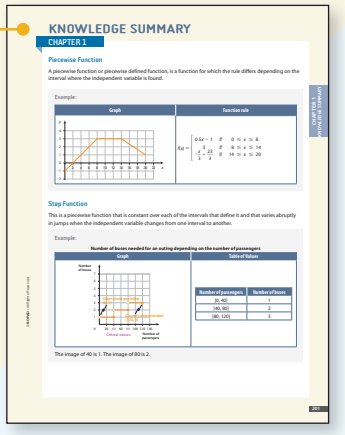
Presented in the first part of the *Complements*, the *Self-Evaluation* allows the learner to evaluate their acquired knowledge and the mathematical competencies they have developed throughout the course. A self-evaluation grid is also provided.

This is a chance for the learner to determine whether a revision is necessary before they move on to the *Summary Scored Activity*.



REFRESHER

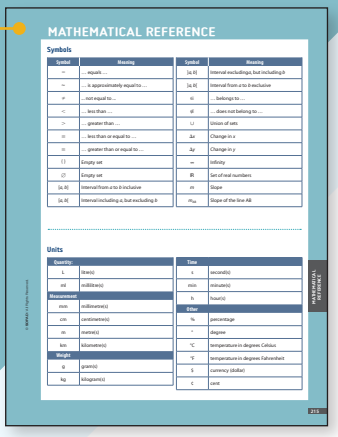
The *Refresher* section uses exercises to review the mathematical rules and concepts that are the subject of a *Reminder*. *Refreshers* may be completed concurrently with learning activities or prior to the course.



KNOWLEDGE SUMMARY

The full version of the *Knowledge Summary* is located in this section.

A printable version is also available online.



MATHEMATICAL REFERENCE

This section presents the mathematical symbols used in the guide, as well as abbreviations of certain units of measurement and reminders of mathematical formulas.

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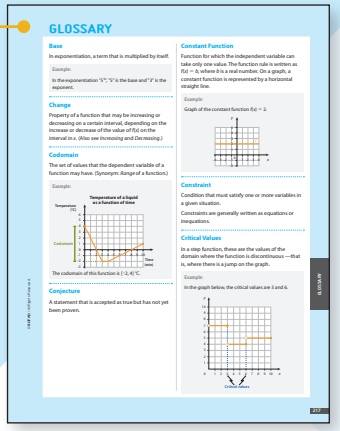
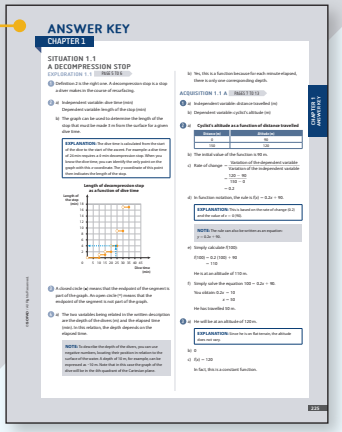


GLOSSARY

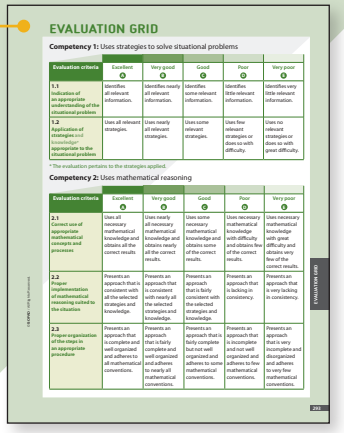
Words and expressions **written in blue** in the current text are defined in the *Glossary*.



ANSWER KEY

The *Answer Key* is designed to allow the learner to check their answers and to complement the learning process.

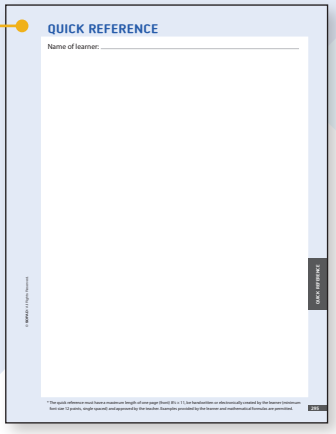
This section contains the answers to questions and detailed explanations of the approach to be taken or the reasoning to be used.



EVALUATION GRID

After solving a *LES*, the learner is asked to evaluate themselves using this grid. Teachers are provided with a list of observable factors on portailsofad.com and in the course teaching guide.

The learner may complete the concise version of the *LES* with their teacher's guidance.



QUICK REFERENCE

A detachable *Quick Reference* page is provided at the very end of the guide. The learner can make notes on this sheet as they progress through the course, and they may refer to it in the certification exam.

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Headings

SCATTER PLOT *

Refers, if applicable, to optional knowledge. It is recognizable by its paler background.

Invites the learner to watch a video clip on the situational problem.

TASK

To validate her friend's assessment of the dive, you must determine…

Presents the task to be performed as part of the situational problem.

REMINDER

REFRESHER EXERCISES
PAGE 195, QUESTIONS 1 TO 2

Representation of…

An interval is a set…

Example:

The interval of the numbers 2…

Refers to knowledge that the learner has acquired in previous courses and refresher exercises related to this *Reminder*.

REMEMBER

Step Functions

A step function has the…

Example:

The interval of the numbers 2…

Presents the mathematical knowledge to be mastered, as prescribed by the study program.

STRATEGY Interpret a…

For a good understanding of a graph, it is essential…

Presents problem-solving strategies that can be applied to a variety of situations.

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Two triangles are congruent if all their corresponding sides are congruent.

Refers to a geometric statement. A complete list is available in the *Mathematical Reference* section.

DID YOU KNOW?

In reality, a pendulum's oscillations (back-and-forth movements) tend to diminish due to…

Allows the learner to discover historical and cultural information related to the mathematical concepts being studied.

TIP

It is not always possible to determine the precise interval of a cycle from the graduations on the axes of a graph...

Provides a tip that simplifies the task, or offers a different way of dealing with the problem or of applying the concept being studied.

CAUTION!

Make sure that the intervals that define the parts of the function in your rule are…

Warns the learner of traps to avoid or exceptions that may apply to the concept being studied.

ICT

In ICT activity 1.2.1, you can observe the cycles and the period of a periodic function. Find this activity on portailsofad.com.

Prompts the learner to complete an online activity (GeoGebra or graphing calculator) that will encourage them to explore the concept studied using technological tools.

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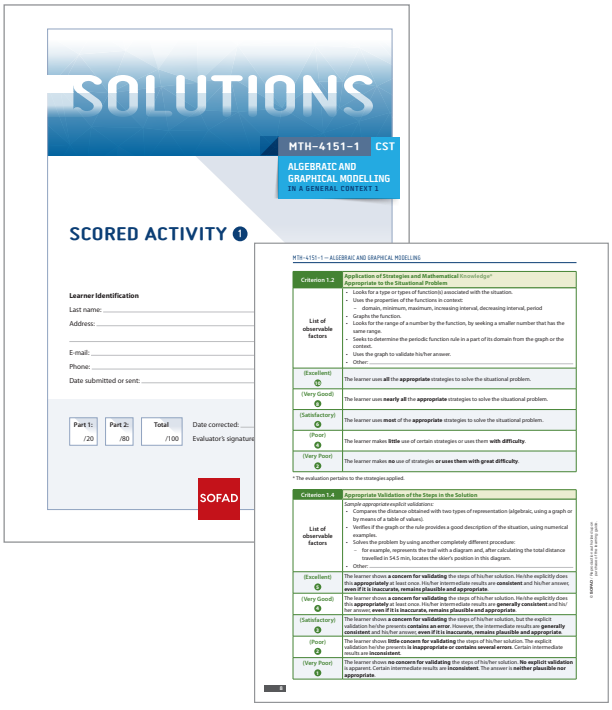
Portailsofad.com for Learners

On **portailsofad.com**, the learner can access:

- A guideline
- Video clips
- ICT activities
  - GeoGebra
  - Graphing calculator
- Printable versions of resources complementary to the **SOLUTIONS** series
  - Scored activities
  - *Knowledge Summary*
  - Complete answer key to the guide in PDF format
  - Lists of observable factors in each *LES*



Scored Activities

The aim of the scored activities is to track the learner's progress. Each learning guide is generally accompanied by two scored activities presented in separate booklets. The learner is expected to complete these scored activities at certain points during the course. These activities make it possible to evaluate the 11 competencies identified by the diversified basic education program. Evaluation grids containing the observable factors are provided with the answer keys. To make it easier to follow, the correction grid contains information about the mathematical knowledge and competencies being evaluated as well as references to the *Situations* in which they were acquired.

SCORED ACTIVITY

You must now complete Scored Activity 1. It can be found on the course website…

In the learning guide, a text box indicates that a scored activity is to be completed after a certain *Situation*.

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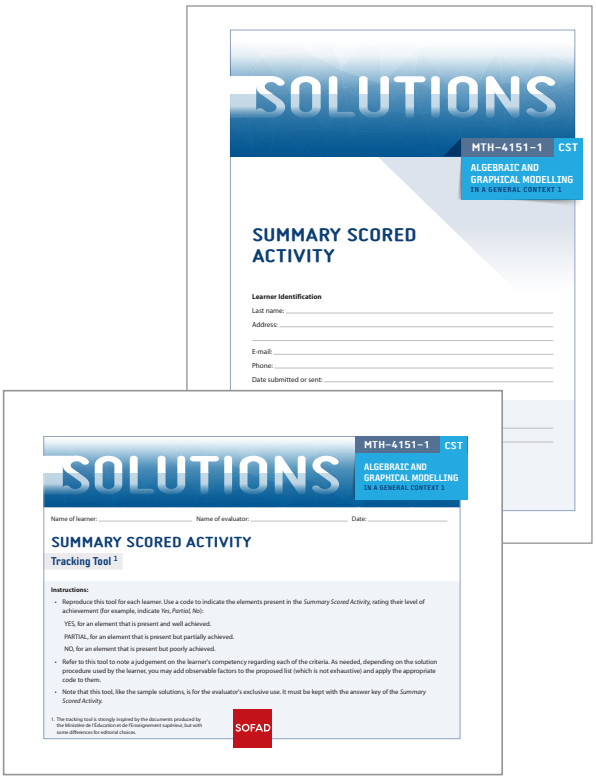
Summary Scored Activity

Each learning guide includes a *Summary Scored Activity*. Also presented separately, the summary scored activity must be completed at the very end of the course. This enables the teacher to assess the extent to which each learner has mastered the mathematical knowledge and competencies before the learner's application to take the ministry exam is considered. In addition to the mathematical knowledge, this activity evaluates the five criteria specified in the *Definition of the Evaluation Domain* (DED). The *Summary Scored Activity* is accompanied by an answer key and a tracking tool.

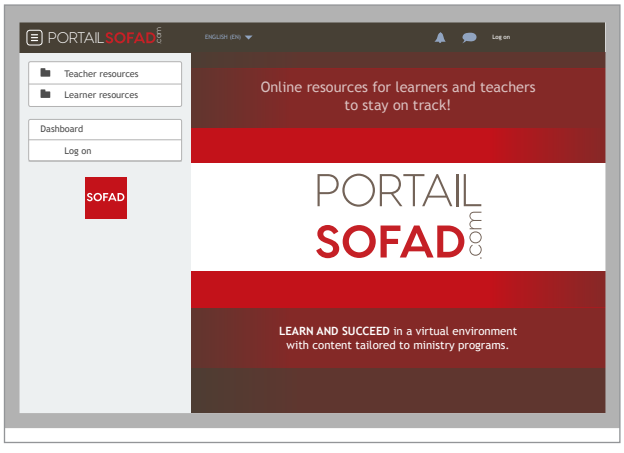


SCORED ACTIVITY

You must now perform the summary scored activity covering all the knowledge of the guide. Find this activity at portailsofad.com.

At the end of the *Self-Evaluation*, a text box indicates that the summary scored activity is to be completed.

All scored activities, and their complements, can be downloaded from the teachers' section of **portailsofad.com**.



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Situational Problems

The *Mathematics Program of Study* defines a situational problem as a complex task that cannot be completed effectively without learning certain concepts or developing problem-solving strategies.

Phases in the Problem-Solving Process

The diagram below illustrates the process of solving a situational problem. These four interrelated phases are described in the program and are echoed in the phases of each *Situation* in the **SOLUTIONS** series.

Phases in the Problem-Solving Process

REPRESENTATION → Exploration
- Statement of the situational problem.
- Task.
- Exploration questions
  - Analyze the situational problem
  - Apply representation strategies.

PLANNING → Acquisition at the start of the Solution
- Identify the relevant knowledge and knowledge to be acquired.
- Apply planning strategies.

ACTIVATION → Solution

Follow the plan devised to solve the problem, taking constraints into account and employing a variety of resources to:
- Verify
- Specify
- Apply activation strategies.

REFLECTION

→ Throughout the learning process

In every phase, the learner must reflect and take risks to solve the situational problem.

Making mistakes is part of the learning process.

The learner must also apply reflection strategies.

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Nature of Learning Activities

The learner acquires mathematical knowledge and develops subject-specific competencies through the situational problems. Each problem is inspired by a context that is meaningful to the learner, who must apply mathematical knowledge or concepts to solve it or draw a conclusion. Situational problems are chosen according to the nature and complexity of the prescribed knowledge and are drawn from the four learning-situation families as described in the mathematics program. The learner then applies the acquired knowledge using procedures incorporated in each course code. In order to put these integrative processes into practice, the learner must develop and deploy subject-specific competencies.

Families of Learning Situations

Each situation falls into one of the program's four mathematical themes, based on certain characteristics, types of problems, or broad issues common to many situations. The learner must solve real-life situational problems that will enable them to build mathematical knowledge and develop subject-specific competencies.

Measurement and Spatial Representation
Situational problems requiring the learner to provide a geometric representation of an object, a physical space, a transformation or a geometric locus.

Relationship Between Quantities
Situational problems requiring the learner to use a graphical or algebraic model that expresses a relation or a dependency relationship between quantities.

Processing Data
Situational problems requiring the learner to collect, compare and process data.

Optimizing Solutions
Situational problems requiring the learner to maximize a profit, a process, or a number of objects or people, or to minimize costs or losses.

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SUBJECT–SPECIFIC COMPETENCIES

The evidence teachers must look for to determine that a learner has developed the three mathematical competencies targeted by the program is presented below.

Subject-Specific Competencies, Key Features and Manifestations

1 Uses Strategies to Solve Situational Problems

1. Defines the problem
- Reformulates the situational problem in his/her own words.
- Identifies the task to be carried out.
- Represents the situational problem mentally or in writing.
- Determines the key elements to be considered and the obstacles to be overcome.
- Selects observation techniques or tools.

2. Searches for possible solutions
- Makes connections.
- Uses lists, tables, diagrams, concrete materials or drawings.
- Refers to the solution of a similar situational problem.
- Uses brainstorming techniques.

3. Chooses a solution
- Takes the constraints into account.
- Takes the consequences into account.
- Takes his/her aptitudes into account.
- Determines the best relationship between the constraints and the consequences.

4. Implements the solution
- Proceeds by trial and error.
- Reviews his/her work.
- Refers to the solutions of a similar situational problem.
- Breaks down a complex situational problem into subproblems.
- Simplifies the situational problem.
- Establishes a plan of action.
- Carries out the plan of action.

5. Validates the solution
- Verifies his/her solution by using examples or counterexamples.
- Compares his/her results to the expected results.
- Compares his/her solution and results to those of others.
- Ensures that his/her solution makes sense.

Source: *Diversified Basic Education Program*, pages 21, 24 and 27.

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2 Uses Mathematical Reasoning

1. Explores the situational problem
- Examines the situational problem.
- Describes the characteristics of the situational problem.
- Asks questions about the situational problem.
- Gathers information about the situational problem.

2. Makes a conjecture
- Proposes probable or plausible ideas.
- Predicts the implications of the ideas proposed.
- Uses examples to find invariants.
- Makes a conjecture.

3. Constructs and uses networks of mathematical cognitive resources
- Establishes organized and functional relationships between different types of knowledge (by associating, classifying, ordering, etc.).
- Uses different forms of representation.
- Selects relevant information.
- Refers to similar situational problems.
- Finds additional information.

4. Draws a conclusion
- Finds examples to verify the conjecture.
- Finds counterexamples to clarify, adjust or refute the conjecture.
- Generalizes by deriving laws, rules or properties.
- Deduces a proposition.

3 Communicates by Using Mathematical Language

1. Decodes the elements of mathematical language
- Recognizes codes and rules.
- Recognizes the meaning of symbols, terms and notation.
- Distinguishes between the mathematical and everyday meaning of various terms.
- Consults different sources of information.

2. Interprets a mathematical message
- Makes connections between the elements of the message.
- Distinguishes between elements that are relevant and those that are not.
- Identifies the key elements of the message.
- Identifies the subject of the message.
- Determines the overall meaning of the situational problem.
- Associates images, objects or knowledge with mathematical terms and symbols.
- Switches from one register of representation to another.
- Verifies his/her understanding of the message.

3. Produces a mathematical message
- Determines the subject of the message.
- Observes codes and rules.
- Uses symbols, terms and notation in accordance with their meaning.
- Uses a register of representation.
- Organizes the message.
- Consults different sources of information.

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PROGRAM OPTIONS

The diversified basic education program offers three distinct options enabling learners to choose the approach to mathematics that best suits their aspirations, interests and aptitudes.

The three Secondary IV options focus on different needs and relate to the following areas: *Cultural, Social and Technical (CST)*, *Technical and Scientific (TS)* and *Science (Sci)*.

Each of the three options prepares learners for postsecondary studies and may also lead to trades, occupations or technical fields that can be studied at the secondary or college level. The following table profiles the three options offered in the program of study.

Table Describing the Three Secondary IV Options

CULTURAL, SOCIAL AND TECHNICAL (CST)	TECHNICAL AND SCIENTIFIC (TS)	SCIENCE (SCI)
MTH-4151-1, MTH-4152-2, MTH-4153-2	**MTH-4161-2 , MTH-4162-2, MTH-4163-2**	**MTH-4171-2 , MTH-4172-2, MTH-4173-2**
• Intended for learners who like to design objects and activities, develop projects or participate in bringing them to fruition. • Likely to stimulate interest in social causes and develop entrepreneurial spirit.	• Intended for learners who wish to explore learning situations that sometimes involve both manual and intellectual work.	• Intended for learners who want to understand the cause and mechanism of phenomena, and to explain and make decisions about them. • Learners discover how to develop formal proofs in learning situations where there is a need to confirm a truth.
Focused on situations that learners are likely to encounter in their personal and professional lives.	Focused on case studies and learners' ability to identify errors and anomalies in solutions with a view to defining the problem and taking corrective action.	Focused on finding, developing and analyzing models primarily in relation to scientific experiments.
Seeks to bring together aspects of mathematics that will help learners become autonomous citizens who play an active role in society. The content allows learners to build on and enrich their basic mathematical knowledge.	Seeks to enable learners to identify the mathematical concepts and processes associated with the design, operation or use of certain technical instruments.	Seeks to develop learners' capacity for abstract thinking by focusing on the properties of mathematical objects, given the complexity of the algebraic operations they encounter.
Prepares learners more specifically to pursue studies in the arts, communications, humanities and social sciences.	Prepares learners more specifically to work effectively in technical fields related to nutrition, biology, physics, business administration and graphic arts.	Prepares learners to pursue studies in the natural sciences or to specialize in research.

Source: *Diversified Basic Education Program*, page 60.

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The tables below describe the prescribed knowledge for the Secondary IV program and indicate the option or options to which they apply.

Algebraic and Graphical Modelling

MATHEMATICAL KNOWLEDGE	MTH-4151-1 (CST)	MTH-4161-2 (TS)	MTH-4171-2 (SCI)
Operations on numerical and algebraic expressions			
Solving one-variable equations and inequalities: second-degree, square root, exponential, logarithmic (including the properties of radicals, exponents and logarithms)		X	X
Operations on numerical and algebraic expressions (multiplying and dividing polynomials, simplifying rational expressions, numbers expressed using rational exponents, radicals and the powers of base 2 and base 10)		X	X
Constructing and interpreting tables of values consisting of positive rational numbers written in base 2 and base 10 (exponential and logarithmic forms)		X	
Expanding, factoring (factoring by grouping and using second-degree algebraic identities, including the perfect square trinomial and the difference of two squares)		X	X
Factoring trinomials using roots			X
Completing the square			X
Relation, function and inverse			
Experimenting with real functions as well as observing, interpreting, describing and representing them (second-degree polynomial, exponential, periodic, step, piecewise)	X		
Experimenting with real functions as well as observing, interpreting, describing and representing them (second-degree polynomial, exponential, square root, periodic, step, logarithmic, greatest integer, piecewise)		X	
Experimenting with real functions as well as observing, interpreting, describing and representing them (second-degree polynomial, step, greatest integer)			X
Describing and interpreting the properties of real functions using a graph	X	X	X
Interpreting the multiplicative parameter		X	
Interpreting multiplicative and additive parameters			X
Solving and graphing two-variable first-degree inequalities		X	
Switching from one form to another in writing second-degree polynomial functions			X
System			
Representing a situation using straight lines	X		
Representing a situation using straight lines or half-planes		X	X
Solving systems of two-variable first-degree equations	X	X	X
Solving systems composed of a first-degree equation and a second-degree equation with two variables			X

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DATA COLLECTION

MATHEMATICAL KNOWLEDGE	MTH-4152-1 (CST)	MTH-4162-2 (TS)	MTH-4172-2 (SCI)
One-variable distribution			
Determining and interpreting measures of position and dispersion: - Percentile rank - Mean deviation - Standard deviation	 X X	 X X	
Representing statistical data related to a population or a sample (Stem-and-Leaf Plot)	X		
Two-variable distribution			
Constructing and interpreting two-variable distributions	X	X	X
Graphing a scatter plot	X	X	X
Representing the regression line by means of a rule or graph	X		
Representing and determining the equation of the regression line or curves related to the functional models being studied		X	
Representing and determining the equation of the regression line			X
Interpolating or extrapolating using the regression line	X	X	X
Approximating and interpreting the correlation coefficient	X	X	
Interpreting a correlation qualitatively and quantitatively	X	X	X
Interpolating and extrapolating using the functional model most appropriate to the situational problem		X	X
Probability			
Calculating and interpreting mathematical expectation		X	
Calculating probabilities using statistical data		X	
Representing and determining conditional probability		X	
Determining the odds *for* or the odds *against*		X	
Changing the value of parameters or conditions		X	
Distinguishing between mutually exclusive, nonmutually exclusive, independent and dependent events		X	

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Geometric Representation

MATHEMATICAL KNOWLEDGE	MTH-4153-1 (CST)	MTH-4163-2 (TS)	MTH-4173-2 (SCI)
Metric and trigonometric relations in triangles			
Determining the slope, measurements and positions using metric and trigonometric relations in triangles	X	X	X
• Angles of a triangle	X	X	X
• Angles in a triangle or in figures that can be divided into triangles			X
• Altitude relative to the hypotenuse	X	X	X
• Projection of the leg on the hypotenuse		X	X
• Sides of a triangle	X	X	X
• Area of a triangle and a quadrilateral	X		
• Area of a triangle		X	
• Area and volume of figures			X
• Coordinates of a point of division	X	X	
• Length of a segment	X	X	X
• Length of a segment resulting from a congruence or a similarity			X
• Perpendicular bisector of a segment		X	
• Distance (between two points)	X	X	X
• Areas of triangles, given the measure of an angle and the lengths of two sides or given the measures of two angles and the length of one side		X	
Representing and interpreting situations using triangles:	X	X	X
• Trigonometric ratios (sine, cosine and tangent)	X	X	X
• Sine law	X		X
• Law of cosines			X
• Heron's formula	X		
• Other relations in triangles, specified in the course's list of principles.	X	X	X
Describing the properties of trigonometric ratios	X	X	
Similar and congruent triangles			
Determining the minimum conditions required to conclude that triangles are congruent or similar	X	X	X
Equivalent figures			
Determining measurements			X

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Evaluation Criteria

The learners' progress in mathematics is evaluated based on targeted criteria. After the scored activities, a co-evaluation with each learner according to the 11 criteria described in the program document is recommended. This joint exercise is a chance for the learner to better understand the criteria used to evaluate the extent to which they have mastered the competencies. With respect to the ministry's evaluation for certification purposes, a smaller number of criteria—generally five—may be used to determine the learner's success. Teachers may consult the document containing the *Definition of the Evaluation Domain (DED)* to find out which criteria are used for each ministerial exam.

EVALUATION CRITERIA FOR THE COMPETENCIES TARGETED BY THE COURSE
Competency 1: Uses Strategies to Solve Situational Problems
Indication of an appropriate understanding of the situational problem
Application of strategies and mathematical knowledge appropriate to the situational problem
Development of an appropriate solution*
Appropriate validation of the steps** in the solution
* The solution includes a procedure, strategies and a final answer. ** The mathematical model, operations, properties or relations involved.
Competency 2: Uses Mathematical Reasoning
Formulation of a conjecture suited to the situation
Correct use of appropriate mathematical concepts and processes
Proper implementation of mathematical reasoning suited to the situation
Proper organization of the steps in an appropriate procedure
Correct justification of the steps in an appropriate procedure
Competency 3: Communicates by Using Mathematical Language
Correct interpretation of a mathematical message
Production of a message in keeping with the terminology, rules and conventions of mathematics, and appropriate to the context

Source: *Diversified Basic Education Program, Mathematics.*

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Introduction

The next part of this teaching guide provides important information to be used in planning each course in the mathematics diversified basic education program.

- Summary of the course program
- Introduction to the learning guide
- Integrative processes targeted by the course
- Family of learning situations in the course
- End-of-course outcomes
- Prescribed mathematical knowledge
- Overall structure of the guide
- Structure of each chapter and its situational problems
- Details about each end-of-chapter LES and the tracking tools used
- Strategies for solving situational problems
- Structure of ICT activities
- Structure of evaluation activities

Conclusion

This *Introduction* to the summary teaching guide provides an overview of the program and its requirements. It outlines a structured learning approach based on solving situational problems that are meaningful to learners.

Each section of the teaching guide that refers to a particular course code in the **SOLUTIONS** series contains specific references, notes and a wealth of useful information for teachers. This helps teachers to plan their work accurately and efficiently and to prepare complementary activities that are relevant to each option proposed by the **SOLUTIONS** series, providing guidance throughout the learning process.

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SOLUTIONS

MTH–4151–1 CST

ALGEBRAIC AND GRAPHICAL MODELLING
IN A GENERAL CONTEXT 1

Table of Contents

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Introduction

Welcome to the course *Algebraic and Graphical Modelling in a General Context 1*. In this course, learners will encounter various situations that put the dependency relationships between quantities into context. They will study five new real functions:

- piecewise functions
- step functions
- periodic functions
- quadratic functions
- exponential functions.

They will complete their learning by expanding their knowledge of:

- the properties of straight lines in a Cartesian plane
- solving systems of equations.

Integrative Processes

To work through the learning situations effectively, learners will become accustomed to applying three processes that encourage the acquisition of mathematical knowledge and the subject-specific competencies associated with the course *Algebraic and Graphical Modelling in a General Context 1*.

Integrative Processes

- **Using an algebraic or graphical model to represent a situation**
- **Using an algebraic or graphical model to generalize a set of situations**
- **Interpolating or extrapolating from an algebraic or graphical model**

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Family of Learning Situations

In the course *Algebraic and Graphical Modelling in a General Context 1*, learners encounter learning situations belonging to the ***Relationship Between Quantities*** family. For instance, the situational problems presented in this guide enable learners to:

- determine the number of solutions to a system of equations
- establish dependency relationships between quantities
- extrapolate results using an algebraic rule or a graph.

End-of-Course Outcomes

By the end of this course, learners will be able to:

- interpret and represent a situation
- formulate conjectures, and construct and use models to draw conclusions
- determine the most accurate dependency relationship between quantities (quadratic, exponential, step model or other), even if this does not exactly express the reality observed
- choose the most suitable type of representation (table of values, graph or algebraic equation) to use to study a function and represent its results
- use an algebraic or graphical model to interpolate or extrapolate results
- use an algebraic or graphical model to generalize a set of situations
- clearly structure their approach
- communicate in accordance with mathematical rules and conventions
- validate their solutions
- acknowledge weaknesses in their models by identifying nuances that differentiate them from the reality observed.

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Knowledge Covered in the MTH-4151-1 Course

MATHEMATICAL KNOWLEDGE	RESTRICTIONS AND CLARIFICATIONS	IN THE *SOLUTIONS* GUIDE	
RELATION, FUNCTION AND INVERSE		**SECTION**	**PAGES**
Experimenting with real functions as well as observing, interpreting, describing and representing them	The real functions studied in this course are:		
	• second-degree polynomial function $f(x) = ax^2$	Acquisitions 2.1 A and B	64 to 85
	• exponential function $f(x) = ab^x$ where $a \neq 0$ and $b > 0$	Acquisitions 2.1 A and B	64 to 85
	• periodic functions	Acquisitions 1.2 A and B	28 to 48
	• step functions	Acquisitions 1.1 A and B	4 to 27
	• piecewise functions.	Acquisitions 1.1 A and B	4 to 27
	Functions may be represented using:		
	• a table of values	Chapters 1 and 2	2 to 121
	• an algebraic rule	Chapters 1 and 2	2 to 121
	• a graph, with or without the use of technology.	Chapters 1 and 2	2 to 121
Describing and interpreting the properties of real functions using a graph	The properties of real functions covered in this course are:		
	• domain and codomain (range)	Chapters 1 and 2	2 to 121
	• increasing and decreasing intervals	Chapters 1 and 2	2 to 121
	• extrema	Chapters 1 and 2	2 to 121
	• sign	Chapters 1 and 2	2 to 121
	• x- and y-intercepts.	Chapters 1 and 2	2 to 121
SYSTEM		**SECTION**	**PAGES**
Representing a situation using straight lines	The properties of the following lines are studied:		
	• parallel lines	Acquisition 3.1 B	136 to 143
	• intersecting lines	Acquisition 3.1 B	136 to 143
	• coincident lines	Acquisition 3.1 B	136 to 143
	• perpendicular lines.	Acquisition 3.1 B	136 to 143
	The equation of the line in standard form:		
	• $f(x) = ax + b$	Acquisition 3.1 B	136 to 143
Solving systems of first-degree equations with two variables	Systems of equations may be solved using:		
	• a table of values	Acquisitions 3.1 A and B	124 to 147
	• an algebraic method (of the learner's choice)	Acquisitions 3.1 A and B	124 to 147
	• a graphical method, with or without the use of technology.	Acquisitions 3.2 A and B	148 to 168

Source: *Mathematics Adult General Education Program; Diversified Basic Education (DBE)*, MEES 2017, pp. 127-128.

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CHAPTER 1 — 8 h 20 min

PIECEWISE, STEP AND PERIODIC FUNCTIONS

Modelling Physical Activity

Broad area of learning:
Health and Well-Being

SITUATION 1.1

pp. 4 to 27

- Piecewise functions
- Step functions

SP – A DECOMPRESSION STOP ▸1.1 **p. 4**

EXPLORATION 20 min

ACQUISITION A 40 min

- Representing a piecewise function
- Interpreting a piecewise function
- Interpreting a step function from its graph

SOLUTION 15 min

ACQUISITION B 40 min

- Expressing a situation as a step function
- Determining and interpreting the properties of piecewise and step functions

CONSOLIDATION 60 min

SITUATION 1.2

pp. 28 to 48

- Periodic functions

SP – THE EFFECT OF EXERCISE ON THE BODY ▸1.2 **p. 28**

EXPLORATION 20 min

ACQUISITION A 40 min

- Discovering the concepts of cycle and period
- Interpreting periodic functions
- Graphing periodic functions

ICT 1.2.1 **p. 35**

SOLUTION 15 min

ACQUISITION B 40 min

- Determining certain properties of a periodic function
- Extrapolating or interpolating the image of a number from a periodic function

CONSOLIDATION 60 min

KNOWLEDGE SUMMARY (**pp. 49 to 52**) 30 min

INTEGRATION (**pp. 53 to 59**) 90 min

LES: A Training Session (**pp. 60 and 61**) 30 min

CHAPTER 2 — 8 h 20 min

QUADRATIC AND EXPONENTIAL FUNCTIONS

Calculate Before You Buy

Broad area of learning: Environmental Awareness and Consumer Rights and Responsibilities

SITUATION 2.1

pp. 64 to 85

- Quadratic functions

SP – A GIFT FOR A LOVED ONE ▸2.1 **p. 64**

EXPLORATION 20 min

ACQUISITION A 40 min

- Recognizing situations of proportionality to the square
- Representing these situations algebraically and using a graph
- Estimating the value of one variable from another in these situations

ICT 2.1.1 **p. 73**

SOLUTION 15 min

ACQUISITION B 40 min

- Graphing quadratic functions
- Describing the properties of a quadratic function

ICT 2.1.2 **p. 77**

CONSOLIDATION 60 min

SITUATION 2.2

pp. 86 to 108

- Exponential functions

SP – THE REAL COST OF A NEW CAR ▸2.2 **p. 86**

EXPLORATION 20 min

ACQUISITION A 40 min

- Determining and interpreting the rule of an increasing or decreasing exponential function
- Graphing an exponential function.
- Interpreting the graph of an exponential function to estimate the value of a variable

ICT 2.2.1 **p. 97**

SOLUTION 15 min

ACQUISITION B 40 min

- Determining the rule of an exponential function from its graph
- Comparing the graphs of the exponential functions defined by the set of real numbers

ICT 2.2.2 **p. 103**

CONSOLIDATION 60 min

KNOWLEDGE SUMMARY (**pp. 109 to 113**) 30 min

INTEGRATION (**pp. 114 to 119**) 90 min

LES: A Down Payment on a Condo (**pp. 120 and 121**) 30 min

SCORED ACTIVITY **1** Pertains to the knowledge acquired in Chapters 1 and 2.

CHAPTER 3 — 8 h 20 min

SYSTEMS OF EQUATIONS AND STRAIGHT LINES

Managing Your Business Well

Broad area of learning: Career Planning and Entrepreneurship

SITUATION 3.1

pp. 124 to 147

- Solving a system using a graph
- Linear equations
- Relative position of straight lines

SP – MANAGING PRODUCTION ▸3.1 **p. 124**

EXPLORATION 20 min

ACQUISITION A 40 min

- Expressing a situation as a system of first-degree equations with two variables
- Solving a system of equations using a graph

SOLUTION 15 min

ACQUISITION B 40 min

- Determining the equation of a straight line by calculating its slope
- Determining the relative position of two straight lines based on their characteristics

ICT 3.1.1 **p. 140**

CONSOLIDATION 60 min

SITUATION 3.2

pp. 148 to 168

- Solving a system using the elimination method
- Solving a system using the substitution method

SP – REMUNERATION IN RETAIL SALES ▸3.2 **p. 148**

EXPLORATION 20 min

ACQUISITION A 40 min

- Solving a system using the elimination method

SOLUTION 15 min

ACQUISITION B 40 min

- Solving a system by substitution
- Determining the number of solutions of a system of equations
- Choosing a method to solve a system of equations

CONSOLIDATION 60 min

KNOWLEDGE SUMMARY (**pp. 169 to 173**) 30 min

INTEGRATION (**pp. 174 to 177**) 90 min

LES: Subcontracting (**pp. 178 and 179**) 30 min

SCORED ACTIVITY **2** Pertains to the knowledge acquired in Chapter 3.

SELF-EVALUATION Pertains to all the mathematical knowledge acquired during the course, with a self-evaluation grid for the learner's use.

SUMMARY SCORED ACTIVITY Pertains to all the mathematical knowledge acquired during the course, with a tracking tool for the evaluator's use.

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Page numbers refer to the MTH-4151-1 learning guide.

STRUCTURE OF COURSE AND CHAPTERS

MTH-4151-1

STRUCTURE OF COURSE AND CHAPTERS

SUMMARY TEACHING GUIDE

MTH-4151-1 CST

ALGEBRAIC AND GRAPHICAL MODELLING

IN A GENERAL CONTEXT 1

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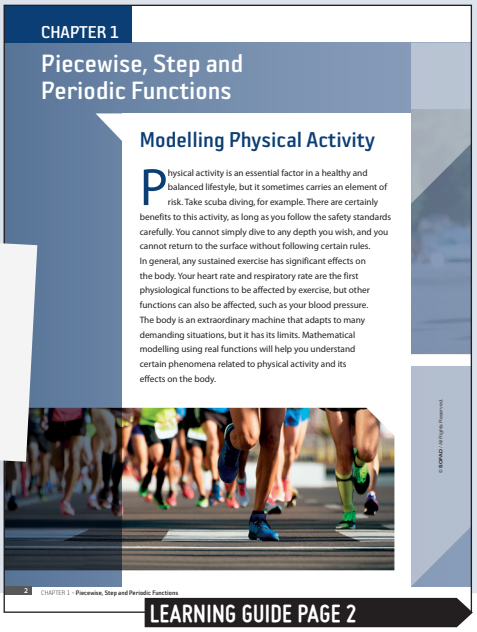
Piecewise, Step and Periodic Functions

Modelling Physical Activity


CHAPTER 1

Piecewise, Step and Periodic Functions

Modelling Physical Activity

Physical activity is an essential factor in a healthy and balanced lifestyle, but it sometimes carries an element of risk. Take scuba diving, for example. There are certainly benefits to this activity, as long as you follow the safety standards carefully. You cannot simply dive to any depth you wish, and you cannot return to the surface without following certain rules. In general, any sustained exercise has significant effects on the body. Your heart rate and respiratory rate are the first physiological functions to be affected by exercise, but other functions can also be affected, such as your blood pressure. The body is an extraordinary machine that adapts to many demanding situations, but it has its limits. Mathematical modelling using real functions will help you understand certain phenomena related to physical activity and its effects on the body.

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2 CHAPTER 1 – Piecewise, Step and Periodic Functions

LEARNING GUIDE PAGE 2

Knowledge to be acquired in this chapter:

- piecewise functions
- step functions
- periodic functions.

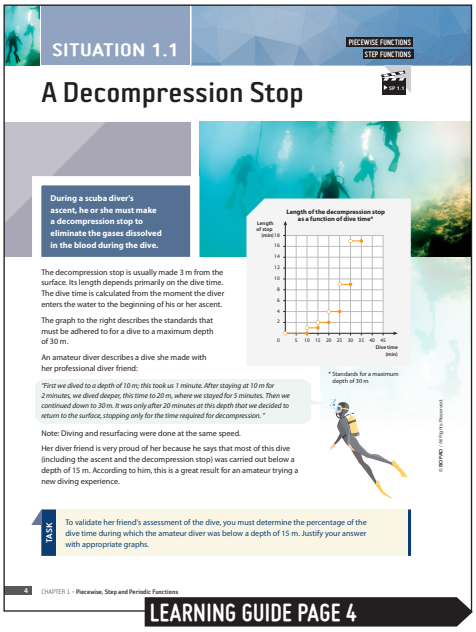
SITUATIONAL PROBLEM 1.1
A Decompression Stop

SITUATION

During a scuba diver's ascent, he or she must make a decompression stop to eliminate the gases dissolved in the blood during the dive.

TASK

Determine the percentage of the dive time during which the amateur diver was below a depth of 15 m. Justify the answer with appropriate graphs.


SITUATION 1.1

PIECEWISE FUNCTIONS
STEP FUNCTIONS

A Decompression Stop

During a scuba diver's ascent, he or she must make a decompression stop to eliminate the gases dissolved in the blood during the dive.

Length of the decompression stop as a function of dive time*

Length of stop (min)

Dive time (min)

The decompression stop is usually made 3 m from the surface. Its length depends primarily on the dive time. The dive time is calculated from the moment the diver enters the water to the beginning of his or her ascent.

The graph to the right describes the standards that must be adhered to for a dive to a maximum depth of 30 m.

An amateur diver describes a dive she made with her professional diver friend:

"First we dived to a depth of 10 m; this took us 1 minute. After staying at 10 m for 3 minutes, we dived deeper, this time to 20 m, where we stayed for 5 minutes. Then we continued down to 30 m. It was only after 20 minutes at this depth that we decided to return to the surface, stopping only for the time required for decompression."

* Standards for a maximum depth of 30 m

Note: Diving and resurfacing were done at the same speed.

Her diver friend is very proud of her because he says that most of this dive (including the ascent and the decompression stop) was carried out below a depth of 15 m. According to him, this is a great result for an amateur trying a new diving experience.

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TASK

To validate her friend's assessment of the dive, you must determine the percentage of the dive time during which the amateur diver was below a depth of 15 m. Justify your answer with appropriate graphs.

4 CHAPTER 1 – Piecewise, Step and Periodic Functions

LEARNING GUIDE PAGE 4

EXPLORATION

Enables learners to review the concepts of variable, interval, function, domain and range as they relate to a function.

STRATEGIES

Use different types of representations to interpret the facts of a problem. (Exploration)

Plan a graph. (Acquisition A)

Plan the solution to a problem. (Solution)

SOLUTION

Interpret a step function from its graph.

Represent a piecewise function.

Interpret a piecewise function.

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SITUATIONAL PROBLEM 1.2
The Effect of Exercise on the Body

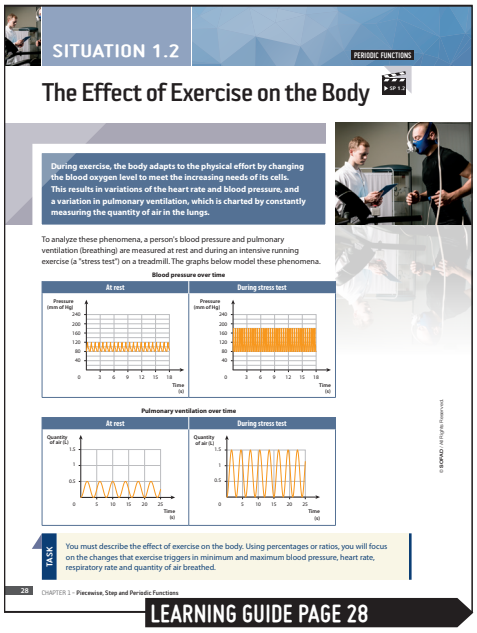
SITUATION

During exercise, the body adapts to the physical effort by changing the blood oxygen level to meet the increasing needs of its cells. This results in variations of the heart rate and blood pressure, and a variation in pulmonary ventilation.

TASK

Describe the effect of exercise on the body.

Using percentages or ratios, focus on the changes that exercise triggers in minimum and maximum blood pressure, heart rate, respiratory rate and quantity of air breathed.



LEARNING GUIDE PAGE 28

EXPLORATION

Helps learners to grasp the context and graphs, and to fully understand the task.

STRATEGIES

Interpret a graph. (Exploration)

Analyze a function based on its properties. (Exploration)

Use a table to make comparisons. (Solution)

SOLUTION

Apply the concepts of cycle and period.

Interpret graphs of periodic functions.

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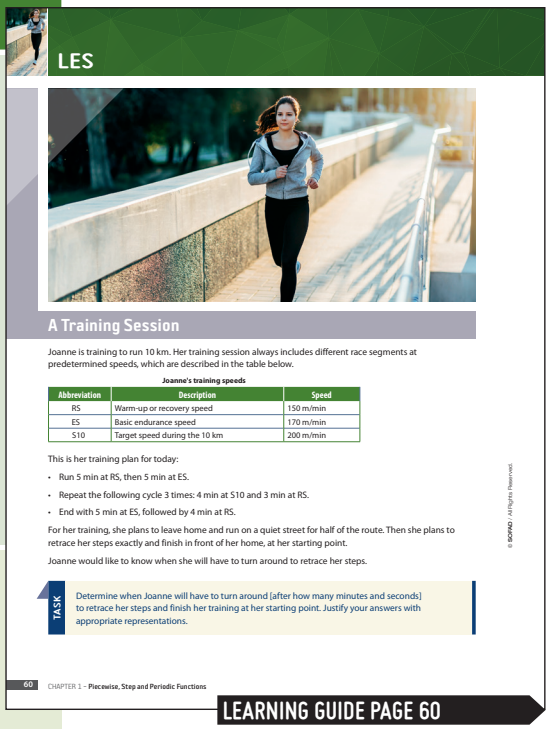
A Training Session

SITUATION

A training session includes different running segments at predetermined speeds. Determine when the athlete will have to turn around and retrace her steps so that she finishes her training at the starting point. Justify your answers with appropriate representations.

LES1
Sample Procedures and Strategies


LES

A Training Session

Joanne is training to run 10 km. Her training session always includes different race segments at predetermined speeds, which are described in the table below.

Joanne's training speeds

Abbreviation	Description	Speed
RS	Warm-up or recovery speed	150 m/min
ES	Basic endurance speed	170 m/min
S10	Target speed during the 10 km	200 m/min

This is her training plan for today:

- Run 5 min at RS, then 5 min at ES.
- Repeat the following cycle 3 times: 4 min at S10 and 3 min at RS.
- End with 5 min at ES, followed by 4 min at RS.

For her training, she plans to leave home and run on a quiet street for half of the route. Then she plans to retrace her steps exactly and finish in front of her home, at her starting point.

Joanne would like to know when she will have to turn around to retrace her steps.

TASK

Determine when Joanne will have to turn around [after how many minutes and seconds] to retrace her steps and finish her training at her starting point. Justify your answers with appropriate representations.

60 CHAPTER 1 – Piecewise, Step and Periodic Functions

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LEARNING GUIDE PAGE 60

REPRESENTATION

Collect all the relevant information in the form of:

- a table
- a list
- a diagram or other form.

Select the most appropriate representation method (algebraic, table of values or graph).

Estimate when Joanne will have to turn around.

Deduce some additional information (for example, distance run) based on the information provided.

Clearly define the information being sought (1: distance and 2: time).

PLANNING

Break the situational problem down into phases:

- Determine the distance at which Joanne is halfway through her run.
- Calculate the time it takes to reach this distance.

ACTIVATION

Follow the plan.

Draw on the mathematical knowledge needed:

- Piecewise functions.
- Linear functions.

Make an approximation to predict the results in terms of distance and time.

Refer to previous situational problems to represent the cumulative distance run as a function of elapsed time and calculate the elapsed time halfway through the run.

REFLECTION

Make sure that the solution makes sense.

Check that all the units and elements of the graph are present.

Consider whether there is a more effective approach.

Compare the result with the anticipated result.

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A Training Session

CRITERIA	OBSERVABLE FACTORS
COMPETENCY 1: Uses Strategies to Solve Situational Problems	
CRITERION 1.1 Indication of an appropriate understanding of the situational problem	☐ Recognizes the requirement to determine when Joanne will have to turn around so that she finishes her training at the starting point. ☐ Takes into account the fact that the training plan includes different phases. ☐ Takes into account the fact that the cumulative distance run is a function of the elapsed time. ☐ Other:
CRITERION 1.2 Application of strategies and mathematical knowledge appropriate to the situational problem	☐ Looks for a model of the relationship between the cumulative distance run and the elapsed time by one of the following means: – Calculates the distance run in each phase and the cumulative distance run – Graphs the function – Determines which phase corresponds to the halfway point – Establishes the function rule for this phase. ☐ Uses the concepts of piecewise functions and linear functions (rate of change and *y*-intercept). ☐ Other:
COMPETENCY 2: Uses Mathematical Reasoning	
CRITERION 2.1 Correct use of appropriate mathematical concepts and processes	☐ For each phase, the range of the function is calculated based on Joanne's training plan. ☐ The phase in which Joanne reaches the halfway point of her run (3400 m) is determined using a table of values or a graph. ☐ The function rule is determined based on the data provided and from the fifth phase or the graph. (For $17 \leq x \leq 21$, $f(x) = 200x - 550$) ☐ The time elapsed at the halfway point (3400 m) is determined based on the rule (19.75 min). ☐ Other:
CRITERION 2.2 Proper implementation of mathematical reasoning suited to the situation	☐ Recognizes that the situation can be modelled by a piecewise function, with each piece corresponding to a linear function. ☐ Determines the function rule for the phase corresponding to the halfway point of the run. ☐ Interpolates the function using the rule. ☐ Other:
CRITERION 2.3 Proper organization of the steps in an appropriate procedure	☐ The reasoning steps are presented clearly. ☐ The representation of the function (whether graphical, algebraic or by means of a table of values) complies with mathematical rules and conventions. ☐ The use of mathematical symbols is appropriate. ☐ The answer is consistent with the procedure. ☐ The answer takes the context into account.

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SOLUTIONS

The SOLUTIONS series covers all the courses in the Diversified Basic Education (DBE) program, including the *Secondary IV Cultural, Social and Technical (CST)* option.

The summary teaching guide presents the learning approach used in the **SOLUTIONS** series, which is based on the acquisition of all the prescribed mathematical knowledge in a problem-solving context.

The summary teaching guide also provides an overview of the three courses offered in the *Secondary IV Cultural, Social and Technical (CST)* option. Designed as a reference for teachers, this support document provides a summary of the elements of the Diversified Basic Education (DBE) program such as the nature of the learning activities, families of learning situations, subject-specific competencies, program options, prescribed knowledge and evaluation criteria.

Components of the summary teaching guide covering each learning guide in the SOLUTIONS series:

- Summary of the elements of the CCBE or DBE program.
- Table providing an overview of the course structure.
- Phases in solving situational problems and LES activities.
- List of observable factors for LES activities.
- Strategies for solving situational problems.
- Structure of ICT activities.
- Structure of evaluation activities.

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