

LEARNING GUIDE

MATHEMATICS

DBE

SOLUTIONS

2ND EDITION

MTH-3051-2

ALGEBRAIC AND
GRAPHICAL MODELLING



IN COMPLIANCE
WITH THE NEW
PROGRAM
OF STUDY

SOFAD

LEARNING GUIDE

MATHEMATICS

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MTH-3051-2

ALGEBRAIC AND
GRAPHICAL
MODELLING

Nicole Perreault

2nd edition

SOFAD

ALGEBRAIC AND GRAPHICAL MODELLING

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Introduction

INTRODUCTION

Welcome to the course *Algebraic and Graphical Modelling*. This is the first mathematics course that you are required to take in Secondary III. It is designed to familiarize you with algebra and enable you to deal with situations that need to be represented with an algebraic or graphical model expressing a relation.

Algebra is a part of daily life. We may not always realize it, but almost everything we do involves this branch of mathematics. Surprising though it may seem, algebra's mix of letters and numbers is the basis of many important calculations we make on a daily basis. Chances are you have already made links between the speed of a vehicle and the amount of time it takes to travel a given distance. Or between your weekly salary and the number of hours you worked. Or even between the number of weeks in a diet and how much weight you lost. But did you know that each sentence in these examples can be converted into an algebraic equation made up of numbers representing quantities and letters representing variables? This is the very essence of algebra. With these numbers and letters, we can process information, construct graphs and make informed decisions. The uses of algebra are virtually limitless. By the end of this course, you will be able to use your new mathematical knowledge of algebra to competently deal with situational problems just like these.

You will also be given opportunities throughout this course to develop your cross-curricular competencies, particularly those involving your communication skills and work methods. Knowing how to represent a situation and express it using the appropriate mathematical language is an important tool in our ever-changing world.

We now invite you to explore the five learning situations in this guide and enrich your knowledge of algebra.



ORGANIZATION AND USE OF THE GUIDE

This guide has been designed for individualized learning either in a classroom setting or through distance education. Its content is inspired by social issues or everyday situations that, although sometimes fictional, are nevertheless realistic.

This approach aims to make your learning more enjoyable by encouraging you to:

- become an active participant in the learning process
- further develop your confidence using algebraic operations
- make the most of your own personal experience and knowledge
- reinvest the knowledge you acquire into your own daily life.

Throughout this guide, you will find tools that you can use to verify your level of understanding and to find ways to overcome any difficulties you may be having so that you can continue to progress and succeed in your learning.

If you get stuck on a particular topic, there is also a resource person who can help. Distance education students can get help from their tutor, while students attending an education centre have access to an instructor. He or she can give you tips, advice or strategies to help you get a firm grasp of the material.

Learning Situations

This guide contains **five learning situations** designed to help you learn new concepts and develop your skills. Each learning situation is organized in the same way. It begins with an introduction, including a description of the task you will need to accomplish by the end of the situation. The first exploration activity reviews some of the concepts covered in previous courses. This is an opportunity to go over certain mathematical ideas and operations that will be useful in the upcoming activities.

This is followed by a number of learning activities. Each one deals with a specific subject and includes a series of questions.

Because these questions deal with new concepts, you may not be able to answer all of them correctly. Try to answer these questions to the best of your ability anyway; the correct answers and explanations follow immediately after the questions. It is essential that you try to understand all of the new concepts that are explained. At the end of these explanations is a series of exercises. The answers to these exercises can be found at the end of the guide.

This is followed by a series of integration exercises that review all of the concepts covered in that learning situation. The answers to these exercises can also be found at the end of the guide.

Once you have completed these exercises, you will be ready to move on to the review summary, putting your communication and logical reasoning skills into practice. Each learning situation ends with a List of New Knowledge. This section is very useful for students who like to see all the essential concepts at a glance.

Visual Cues

Your learning will be guided by captions and visual cues throughout the text.

In the **Glossary**

Words and expressions written in blue are defined in the glossary at the end of the guide.

DID YOU KNOW?

The *Did You Know?* captions enhance the mathematical content with interesting facts, however, there will be no questions about them on the final exam.



TIP

Look for the light bulb to find tips to make your work simpler.



CAUTION!

The exclamation point indicates something you should pay special attention to.



REMINDER

Reminder captions are there to jog your memory about concepts covered in previous courses.



REMEMBER

Look for the paper clip for important points to remember as you progress.



LIST OF NEW KNOWLEDGE

The *List of New Knowledge* recaps the essential concepts you have just learned.



The last section of the guide provides a summary of the course content and a self-evaluation to help you determine whether you have a good understanding of the material and are ready to take the final exam. This section also contains the answer key to this evaluation and all the learning situations, as well as the glossary.

Scored Activities

This guide comes with two scored activities in separate booklets. The purpose of these activities is to chart your progress in real terms. It is important to complete them to the best of your ability without any assistance from others. Refer to the Table of Contents to see when each scored activity must be completed. If you did not receive these booklets, you can download them from the SOFAD website at portailsofad.com.

The following table lists the topics that are evaluated by each scored activity and at what point you are required to complete these activities.

| Evaluation Situation | Topics Covered | To Be Completed |
|----------------------|------------------------------------------------------------------------|----------------------------|
| Scored Activity 1 | Relations (Learning Situation 1 and 2) | After Learning Situation 2 |
| Scored Activity 2 | Inequalities, relations and systems (Learning Situation 3, 4 and 5) | After Learning Situation 5 |

Once you have completed a scored activity, you must submit it to your tutor or instructor for correction. As a general rule, only one scored activity may be submitted at a time. You must also wait for it to be corrected before submitting the next scored activity. You can contact your education centre or school board for more information on this.

Self-Evaluation

The final activity in the guide is a self-evaluation, which gives you the opportunity to assess the knowledge you have acquired and the skills you have developed. The self-evaluation grid that accompanies this activity will help you identify which concepts you have mastered and which ones you should review before taking the certification exam. The grid indicates which activities to review for each concept.

Before completing the self-evaluation, take some time to prepare. Review the concepts found in the *List of New Knowledge* sections and make sure you have completed all of the exercises correctly. It is recommended that you complete the self-evaluation without referring to the guide or answer key. After you have finished this activity, you can compare your answers to those in the answer key and do further review as needed.

Answer Key

An answer key for all of the exercises in this learning guide is included after the self-evaluation activity. Refer to it after each set of exercises to make sure you have fully understood all of the concepts before continuing on to the next activity or learning situation. At the end of this section is the answer key for the self-evaluation activity.

Note that there is no answer key for the questions related to the explanation of new concepts. Only the numbered exercises are included in the answer key.

Glossary

At the very end of this learning guide is a glossary. It gives the definitions of the words written in blue that appear throughout the learning situations. These words are listed in alphabetical order. Refer to the glossary often as these definitions will help you better understand these important terms and expressions.

Additional Materials

Ensure you have all the materials you need.

- In addition to your guide, use a notebook to write down important information and concepts.
- You should also have a dictionary, a calculator, a pencil for writing answers and notes in your guide, a colour pen to correct your answers, a highlighter to emphasize key concepts, an eraser, etc. For certain tasks, you may need certain geometry instruments, such as a ruler in centimetres, a protractor, a set square and a compass.

Learning Support

Whether you are learning at an education centre or through distance education, you are never on your own in the learning process. Students learning in a classroom can get help from their instructor, while distance education students can count on their tutor for support.

Additional Information About Distance Education

The course requires an estimated 75 hours of work. Here are a few suggestions to help you manage your time.

- Once you have received your course materials, create a study schedule based on your family and work obligations as well as the course requirements.
- Try to devote a few hours a week to your studies, preferably setting aside two hours at a time.
- Stick to your schedule as much as possible.

Your tutor is your resource person who will be correcting and commenting on your two scored activities. Do not hesitate to consult your tutor if you are having any problems with the theory or exercises, or if you simply need a little encouragement to keep going. Jot down any questions as they arise and contact your tutor by phone during his or her available hours. You can also send your questions by email at any time. If you did not receive your tutor's schedule and contact information with this guide, you can obtain it from the learning centre.

Your tutor is there to guide you throughout your learning process and provide you with the information you need to ensure your success.

Evaluation for Certification Purposes

In order to earn the credits for this course, you must obtain a mark of at least 60% on the final examination, which will be held in an adult education centre. To be eligible to write this examination, ideally you should have obtained an average of at least 60% on the scored activities that accompany this guide. Note that some adult education centres require students to achieve an average of 60% or more on the scored activities in order to take the final examination.

Consult your instructor or tutor for more information about these requirements and to find out when and where the final examination will be held. He or she can also tell you what materials you are allowed to have with you during the examination.

Essential Knowledge in Learning Situations

This course is designed to help you acquire the following mathematical knowledge.

| Learning Situation | Essential Knowledge |
|---------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Variables That Have an Impact | <ul style="list-style-type: none">• Observing, describing, interpreting and representing the dependency between the variables of a situation• Using a scatter plot to represent an experiment or statistical study |
| 2. Functions and Their Inverse: Talk About a Challenge! | <ul style="list-style-type: none">• Exploring the terms relation, function and inverse• Representing and interpreting the inverse of a function• Describing the properties of a function in context |
| 3. A Rule is a Rule | <ul style="list-style-type: none">• Determining the rule of correspondence• Providing a qualitative description of how the graph is affected by a change in the value of a parameter of an affine function |
| 4. Actions for a Changing World | <ul style="list-style-type: none">• Exploring the inequality relation• Solving one-variable first-degree equations and inequalities |
| 5. An Important Meeting | <ul style="list-style-type: none">• Solving systems of two-variable first-degree equations |



Solutions—that is the essence of this mathematics series. Finding solutions means exploring, discovering and learning. It also means reasoning and drawing on previous knowledge to develop new skills.

Algebraic and Graphical Modelling (MTH-3051-2) is the first course in the DBE mathematics program. It consists of five learning situations (LS).

- LS1 Variables That Have an Impact
- LS2 Functions and Their Inverse: Talk About a Challenge!
- LS3 A Rule is a Rule
- LS4 Actions for a Changing World
- LS5 An Important Meeting

SOFAD

ACQUIRE KNOWLEDGE AND DEVELOP SKILLS

THE LEARNING SITUATIONS IN THIS GUIDE WILL ENABLE YOU TO:

- represent and interpret the dependency between the variables of a situation
- represent situations using various types of graph
- solve systems of first-degree equations in one or two variables

OUR LEARNING TOOLS INCLUDE:

- theoretical and practical activities
- numerous exercises and a detailed answer key
- scored activities enabling teachers to track learners' progress

TITLES IN THE *SOLUTIONS* SERIES OF THE DBE MATHEMATICS PROGRAM

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MTH-3052-2 Data Collection

MTH-3053-2 Geometric Representation

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