MTH-3001-2

# The Four Operations on Polynomials

sofad

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sofad

This course was produced in collaboration with the Service de l'éducation des adultes de la Commission scolaire catholique de Sherbrooke and the Department of the Secretary of State of Canada.

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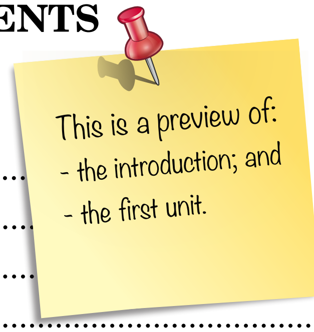





## UNITS

# INTRODUCTION TO THE PROGRAM FLOWCHART

**Welcome to the World of Mathematics!**

This mathematics program has been developed for the adult students of the Adult Education Services of school boards and distance education. The learning activities have been designed for individualized learning. If you encounter difficulties, do not hesitate to consult your teacher or to telephone the resource person assigned to you. The following flowchart shows where this module fits into the overall program. It allows you to see how far you have progressed and how much you still have to do to achieve your vocational goal. There are several possible paths you can take, depending on your chosen goal.

The first path consists of modules MTH-3003-2 (MTH-314) and MTH-4104-2 (MTH-416), and leads to a Diploma of Vocational Studies (DVS).

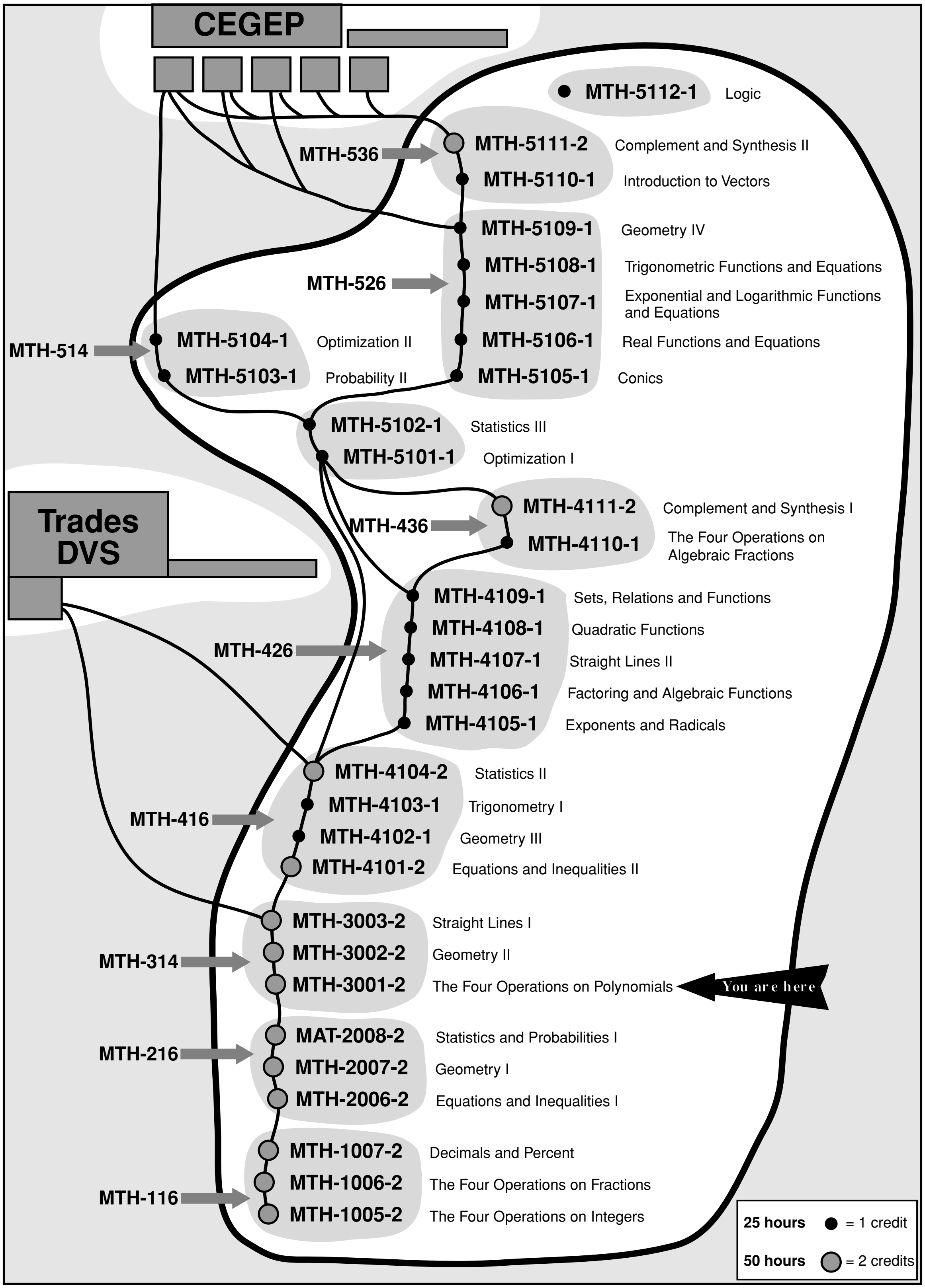
The second path consists of modules MTH-4109-1 (MTH-426), MTH-4111-2 (MTH-436) and MTH-5104-1 (MTH-514), and leads to a Secondary School Diploma (SSD), which allows you to enroll in certain Gegep-level programs that do not call for a knowledge of advanced mathematics.

The third path consists of modules MTH-5109-1 (MTH-526) and MTH-5111-2 (MTH-536), and leads to Cegep programs that call for a solid knowledge of mathematics in addition to other abiliies.

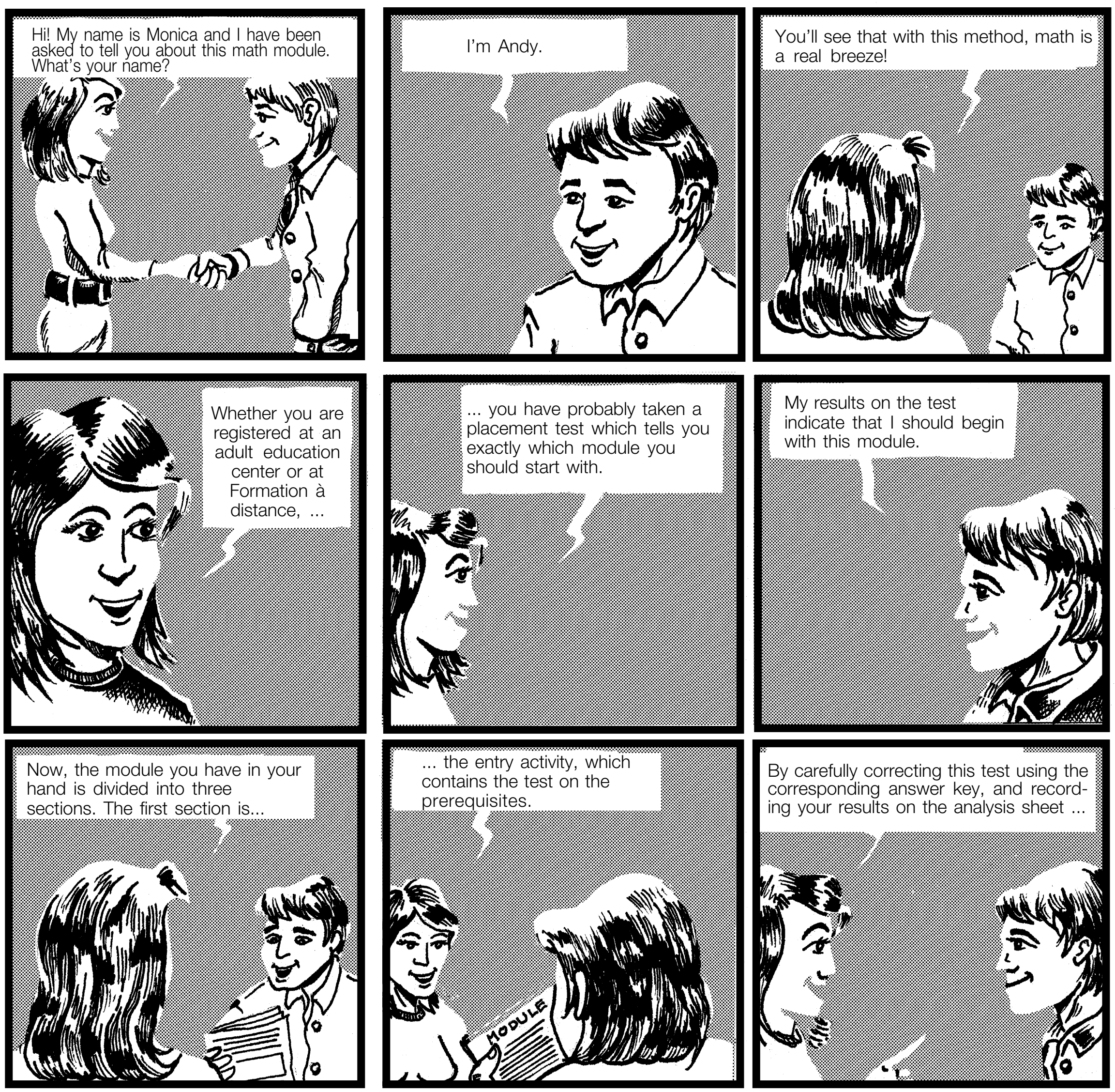
If this is your first contact with this mathematics program, consult the flowchart on the next page and then read the section “How to Use This Guide.” Otherwise, go directly to the section entitled “General Introduction.” Enjoy your work!

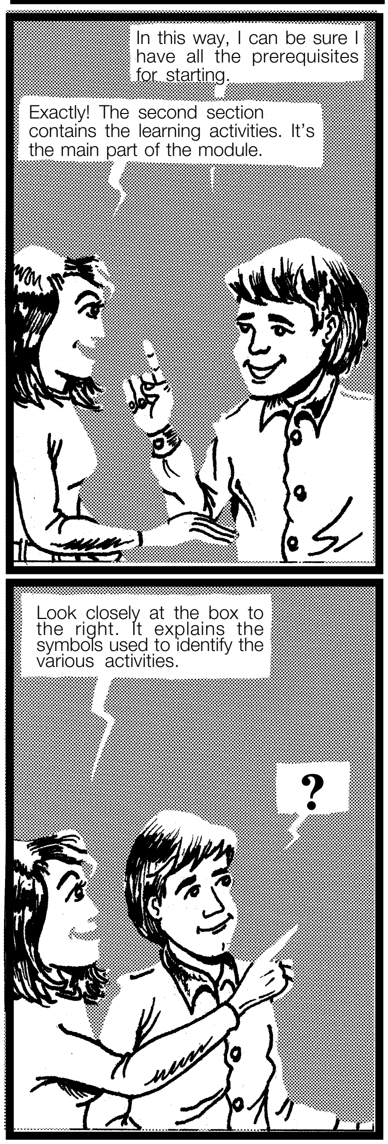
 

# THE PROGRAM FLOWCHART



# HOW TO USE THIS GUIDE




START

The starting line shows where the learning activities **begin**.

The little white question mark indicates the **questions** for which answers are given in the text.

The target precedes the **objective** to be met.

Reminder

The memo pad signals a **brief reminder** of concepts which you have already studied.

?

The boldface question mark indicates **practice exercices** which allow you to try out what you have just learned.

The **calculator** symbol reminds you that you will need to use your calculator.

The sheaf of wheat indicates a **review** designed to reinforce what you have just learned. A row of sheaves near the end of the module indicates the final review, which helps you to interrelate all the learning activities in the module.

FINISH

Lastly, the finish line indicates that it is time to go on to the **self-evaluation test** to verify how well you have understood the learning activities.


There are also many fun things in this module. For example, when you see the drawing of a sage, it introduces a “Did you know that...”
A “Did you know that...”?
Yes, for example, short tidbits on the history of mathematics and fun puzzles. They are interesting and relieve tension at the same time.
Must I memorize what the sage says?
No, it’s not part of the learning activity. It’s just there to give you a breather.
It’s the same for the “math whiz” pages, which are designed especially for those who love math.
They are so stimulating that even if you don’t have to do them, you’ll still want to.
And the whole module has been arranged to make learning easier.
For example. words in boldface italics appear in the glossary at the end of the module...
Great!
... statements in boxes are important points to remember, like definitions, formulas and rules. I’m telling you, the format makes everything much easier.
The third section contains the final review, which interrelates the different parts of the module.
There is also a self-evaluation test and answer key. They tell you if you’re ready for the final evaluation.
Thanks, Monica, you’ve been a big help.
I’m glad! Now, I’ve got to run. See you!
Later ...
This is great! I never thought that I would like mathematics as much as this!

# GENERAL INTRODUCTION

## THE ABC'S OF POLYNOMIALS

Polynomial, monomial, binomial, trinomial ... What do these words mean? Each one is the name of an algebraic expression with specific characteristics. Let us examine the roots of these words: **poly-** and **mono-** come from the Greek and respectively mean **many** and **only one**; the roots **bi-** and **tri-** are of Latin origin and mean **two** and **three**. **Nomial** comes from the Greek and means **name**. However, in the context of an algebraic expression, nomial designates term.

Thus, a polynomial is an expression composed of a term or a sum of terms. But what is a term? It is simply an algebraic expression which is not separated by an addition or subtraction sign. For example, $-3a^2x^4$ and $2y^3$ are terms. There are three types of polynomials which have specific names: the monomial, the binomial and the trinomial. A monomial contains one term, a binomial contains two and a trinomial, three. For example, $2ab^2$ is a monomial, $4x^2y^2 - 2$ is a binomial and $x^2 + 6x + 5$ is a trinomial. We define the expression $8x^3 - x^2 + 4x - 2$ as a polynomial with 4 terms.

Polynomials are extremely useful in mathematics and science. Among other things, they allow us to generalize a situation. For example, the surface area of any triangle can be calculated using the expression $\frac{1}{2}b \times h$, which is a monomial.

Most of the scientific formulas in chemistry, biology and physics are composed of polynomials. In business, the interest on capital is calculated by means of a monomial. In economics, the different upward or downward trends of the market, the cost of living index, or other indexes, are often calculated using polynomials.

It is important then to learn to compute with these algebraic expressions. To do so, certain techniques must be learned. These techniques, while often repetitious and sometimes boring, must be used in order to achieve exact results. We recognize that gymnasts must exercise for many hours on the balance beam, which they do routinely. If you ask them, they will acknowledge that these exercises are tiring, but that perfection has its price. The same goes for mathematics.

In this module, you will learn to distinguish certain mathematical expressions which you must classify as monomials, binomials, trinomials or polynomials.

As you did for the natural numbers ($\mathbb{N}$), the integers ($\mathbb{Z}$) and the rational numbers ($\mathbb{Q}$), you will learn to add, subtract, multiply and divide polynomials. Lastly, you will simplify algebraic expressions containing the four operations (+, –, ÷ and ×) on polynomials. These expressions may contain parentheses, brackets or both and, to simplify them, you will apply a law which you already know: the law of the order of operations.

## INTERMEDIATE AND TERMINAL OBJECTIVES OF THE MODULE

Module MTH-3001-2 contains seven units and requires fifty hours of study distributed as shown below. Each unit covers either an intermediate or a terminal objective. The terminal objectives appear in boldface.

| Objectives | Number of Hours* | % (evaluation) |
|---|---|---|
| 1 to **6** | 48 | 100% |

* Two hours are allotted for the final evaluation.

1. Value of the Numerical Coefficient and the Exponent of a Given Base

   To determine the value of the numerical coefficient of a monomial, as well as the value of the exponent which is affixed to each of the variables (bases) in the monomial. The latter contains at most three variables. In addition, you should be able to recognize similar monomials.

2. Monomials and Polynomials

   To select the monomials and the polynomials from a list containing at most ten algebraic expressions and to order a polynomial according to the increasing or decreasing powers of a given variable.

3. Adding and Subtracting Two Polynomials

   To add or subtract two polynomials, each containing a maximum of four terms; the terms of the polynomials contain a maximum of three variables. The numerical coefficients are rational numbers and the exponents are natural numbers. The polynomial obtained must be in simplified form and its terms must be ordered.

4. Multiplying Two Polynomials

To multiply two algebraic expressions: either a monomial by a monomial, a monomial by a polynomial containing a maximum of three terms, or a binomial by a binomial; the terms have a maximum of three variables. The numerical coefficients are rational numbers and the exponents are natural numbers. If the product obtained is a polynomial, then its terms must be ordered.

5. Dividing Two Polynomials

To divide two algebraic expressions: either a monomial by a monomial, a binomial by a monomial, or a trinomial by a monomial; the terms contain a maximum of three variables. The numerical coefficients are rational numbers, and the exponents (of the divisor, the dividend and the quotient) are natural numbers. If the quotient obtained is a polynomial, then its terms must be ordered.

**6. The Four Operations on Polynomials**

**To simplify an algebraic expression by performing the required operations (addition, subtraction, multiplication or division) and by following the order of operations. The algebraic expression contains a maximum of three sets of parentheses, one set of brackets and ten terms; each term contains a maximum of two variables. The numerical coefficients are rational numbers and the exponents are natural numbers. The exponents affixed to the variables in the simplified form (solution) are all positive. The polynomial obtained must be in simplified form and its terms must be ordered. The steps in the simplification of the algebraic expression must be described.**

# DIAGNOSTIC TEST ON THE PREREQUISITES

## Instructions

1. Answer as many questions as you can.
2. Do not use a calculator.
3. Write your answers on the test paper.
4. Do not waste any time. If you cannot answer a question, go on to the next one immediately.
5. When you have answered as many questions as you can, correct your answers using the answer key which follows the diagnostic test.
6. To be considered correct, answers must be identical to those in the key. For example, if you are asked to describe the steps involved in solving a problem, your answer must contain all the steps.
7. Transcribe your results onto the chart which follows the answer key. It gives an analysis of the diagnostic test results.
8. Do only the review activities listed for each of your incorrect answers.
9. If all your answers are correct, you may begin working on this module.

1. Calculate the sums of the following numbers.

a) $-\frac{3}{2}+\frac{1}{2}$ b) $7.33 + (-2.56)$

2. Calculate the differences between the following numbers.

a) $\frac{7}{8}-\left(-\frac{3}{8}\right)$ b) $12.19 - 11.87$

3. Calculate the products of the following numbers.

a) $\frac{2}{3}\times 3\frac{1}{2}$ b) $4.12(-1.6)$

4. Calculate the quotients of the following numbers.

a) $4\ 760 \div 56$ b) $-4.5 \div 0.9$

c) $-\frac{7}{8}\div\left(-\frac{1}{2}\right)$

5. Evaluate the following expressions by following the order of operations.

a) $4(23 - 12 + 7) \div 9 + 8$ b) $\frac{2}{3}-\frac{1}{3}\left(\frac{1}{2}-\frac{1}{8}\right)$

c) $[(8.5 - 2.5 \times 2.2 - 2.0) + 4.2 \times 1.5] \div 2.3$

6. Monica is a salesperson in a department store which employs 48 women and 29 men. She has been working there for 4 years. She estimates that she sells an average of 14 pairs of shoes per day. If she works 5 days per week and 50 weeks per year, calculate how many pairs of shoes she sells in a year.

7. During a car rally, the Dundees traveled 1 275 km. Ms Dundee drove the first 720 kilometres and Mr Dundee drove the rest of the time. They can drive 425 km on a full tank of gas. If it costs $24.00 to fill up, calculate how much they spent on gas for this rally.

8. Tony is a clerk in a grocery store. He works 4 hours per day, 4 days a week. His salary is one third that of his mother. If she earns $225 per week and if Tony must set aside $35 per week for various expenses, calculate how much he will have saved after 50 weeks of work.

# ANSWER KEY FOR THE DIAGNOSTIC TEST ON THE PREREQUISITES

1. a) $-\frac{3}{2} + \frac{1}{2} = -\frac{2}{2}$ or $-1$

   b) $7.33 + (-2.56) = 4.77$ since $\begin{array}{r} 7.33 \\ -\ 2.56 \\ \hline 4.77 \end{array}$

2. a) $\frac{7}{8} - \left(-\frac{3}{8}\right) = \frac{7}{8} + \frac{3}{8} = \frac{7+3}{8} = \frac{10}{8} = \frac{5}{4}$ or $1\frac{1}{4}$

   b) $12.19 - 11.87 = 0.32$ since $\begin{array}{r} 12.19 \\ -\ 11.87 \\ \hline 0.32 \end{array}$

3. a) $\frac{2}{3} \times 3\frac{1}{2} = \frac{\cancel{2}^{1}}{3} \times \frac{7}{\cancel{2}_{1}} = \frac{14}{6} = \frac{7}{3}$ or $2\frac{1}{3}$

   b) $4.12 \times (-1.6) = -6.592$ since $\begin{array}{r} 4.12 \\ -\ 1.6 \\ \hline 2472 \\ 412 \\ \hline -6.592 \end{array}$

4. a) $4\,760 \div 56 = 85$ since $\begin{array}{r|l} & \ \ 85 \\ \hline 56 & 4760 \\ & 448 \\ \hline & \ \ 280 \\ & \ \ 280 \\ \hline & \ \ \ \ \ 0 \end{array}$

   b) $-4.5 \div 0.9 = -5$ since $\begin{array}{r|l} & \ -5 \\ \hline 9 & -45 \end{array}$

   c) $-\frac{7}{8} \div \left(-\frac{1}{2}\right) = -\frac{7}{\cancel{8}_{4}} \times \left(-\frac{\cancel{2}^{1}}{1}\right) = \frac{14}{8} = \frac{7}{4}$ or $1\frac{3}{4}$

5. a) $4(23 - 12 + 7) \div 9 + 8$

$4(11 + 7) \div 9 + 8$

$4(18) \div 9 + 8$

$72 \div 9 + 8$

$8 + 8 = 16$

b) $\frac{2}{3} - \frac{1}{3}\left(\frac{1}{2} - \frac{1}{8}\right)$

$\frac{2}{3} - \frac{1}{3}\left(\frac{4}{8} - \frac{1}{8}\right)$

$\frac{2}{3} - \frac{1}{3}\left(\frac{3}{8}\right)$

$\frac{2}{3} - \frac{1}{8}$

$\frac{16}{24} - \frac{3}{24} = \frac{13}{24}$

c) $[(8.5 - 2.5 \times 2.2 - 2.0) + 4.2 \times 1.5] \div 2.3$

$[(8.5 - 5.5 - 2.0) + 6.3] \div 2.3$

$[(3.0 - 2.0) + 6.3] \times 2.3$

$[1.0 + 6.3] \times 2.3$

$7.3 \times 2.3 \approx 3.2$

6. • We want to determine how many pairs of shoes Monica sells in one year.
   • State the problem mathematically

   14 pairs/day × (5 days/week × 50 weeks)
   • Estimate the result

   10 pairs/days × (5 days/week × 50 weeks) = 2 500 pairs
   • Solve the problem

   14 pairs/day × (5 days/week × 50 weeks)

   14 pairs/day × 250 days

   3 500 pairs

Monica sells 3 500 pairs of shoes in one year.

7. • We want to determine how much they spent in gas.
   • State the problem mathematically

   (1 275 km ÷ 425 km) × \$24
   • Estimate the result

   (1 200 km ÷ 400 km) × \$20 = \$60

- Solve the problem

  (1 275 km ÷ 425 km) × \$24

  3 × \$24 = \$72

The Dundees spent \$72 on gasoline for the rally.

8. - We want to determine how much Tony will have saved after 50 weeks of work.
   - State the problem mathematically

     $$\left[\left(\frac{1}{3} \times \$225/\text{week}\right) - \$35/\text{week}\right] \times 50 \text{ weeks}$$

   - Estimate the result

     $$\left[\left(\frac{1}{3} \times \$210/\text{week}\right) - \$40/\text{week}\right] \times 50 \text{ weeks}$$

     [\$70/week – \$40/week] × 50 weeks

     \$1 500
   - Solve the problem

     $$\left[\left(\frac{1}{3} \times \$225/\text{week}\right) - \$35/\text{week}\right] \times 50 \text{ weeks}$$

     [\$75/week – \$35/week] × 50 weeks

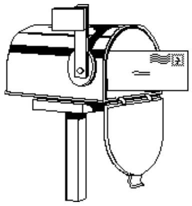
     \$40/week × 50 weeks = \$2 000

Tony will have saved \$2 000 after 50 weeks of work.

# ANALYSIS OF THE DIAGNOSTIC TEST RESULTS

| Question | Answer | | Review | | Before Going to Unit(s) |
|---|---|---|---|---|---|
| | Correct | Incorrect | Section | Page | |
| 1. a) | | | 8.2 | 8.20 | 3 |
| b) | | | 8.2 | 8.20 | 3 |
| 2. a) | | | 8.3 | 8.26 | 3 |
| b) | | | 8.3 | 8.26 | 3 |
| 3. a) | | | 8.4 | 8.32 | 4 |
| b) | | | 8.4 | 8.32 | 4 |
| 4. a) | | | 8.5 | 8.39 | 5 |
| b) | | | 8.5 | 8.39 | 5 |
| c) | | | 8.5 | 8.39 | 5 |
| 5. a) | | | 8.6 | 8.44 | 6 |
| b) | | | 8.6 | 8.44 | 6 |
| c) | | | 8.6 | 8.44 | 6 |
| 6. | | | 8.1 | 8.4 | 1 |
| 7. | | | 8.1 | 8.4 | 1 |
| 8. | | | 8.1 | 8.4 | 1 |

- If all your answers are **correct**, you may begin working on this module.
- For each **incorrect** answer, find the related section listed in the **Review** column and complete it before beginning the unit(s) listed in the right-hand column under the heading **Before Going on to Unit(s)**.



# INFORMATION FOR DISTANCE LEARNING STUDENTS

You now have the learning material for the course MTH-3001-2 as well as the homework assignments and a letter of introduction from your tutor. This letter outlines the various ways for getting in touch with him or her. In addition to correcting your assignments, your tutor is the resource person who will help you with your studies. Do not hesitate to contact your tutor if you have any questions.

## DEVELOPING EFFECTIVE STUDY HABITS

Distance learning is a process which offers considerable flexibility, but which also requires active involvement on your part. It demands regular study and sustained effort. Efficient study habits will simplify your task. To ensure effective and continuous progress in your studies, it is strongly recommended that you:

- draw up a study timetable that takes your working habits into account and is compatible with your leisure time and other activities;

- develop a habit of regular and concentrated study.

The following guidelines concerning the theory, examples, exercises and assignments are designed to help you succeed in this mathematics course.

**Theory**

To make sure you thoroughly grasp the theoretical concepts:

1. Read the lesson carefully and underline the important points.

2. Memorize the definitions, formulas and procedures used to solve a given problem, since this will make the lesson much easier to understand.

3. At the end of an assignment, make a note of any points that you do not understand. Your tutor will then be able to give you pertinent explanations.

4. Try to continue studying even if you run into a particular problem. However, if a major difficulty hinders your learning, ask for explanations before sending in your assignment. Contact your tutor, using the procedure outlined in his or her letter of introduction.

**Examples**

The examples given throughout the course are an application of the theory you are studying. They illustrate the steps involved in doing the exercises. Carefully study the solutions given in the examples and redo them yourself before starting the exercises.

## Exercises

The exercises in each unit are generally modelled on the examples provided. Here are a few suggestions to help you complete these exercises.

1. Write up your solutions, using the examples in the unit as models. It is important not to refer to the answer key found on the coloured pages at the end of the module until you have completed the exercises.

2. Compare your solutions with those in the answer key only after having done all the exercises. **Careful!** Examine the steps in your solution carefully even if your answers are correct.

3. If you find a mistake in your answer or your solution, review the concepts that you did not understand, as well as the pertinent examples. Then, redo the exercise.

4. Make sure you have successfully completed all the exercises in a unit before moving on to the next one.

## Homework Assignments

Module MTH-3001-2 contains three assignments. The first page of each assignment indicates the units to which the questions refer. The assignments are designed to evaluate how well you have understood the material studied. They also provide a means of communicating with your tutor.

When you have understood the material and have successfully done the pertinent exercises, do the corresponding assignment immediately. Here are a few suggestions.

1. Do a rough draft first and then, if necessary, revise your solutions before submitting a clean copy of your answer.

2. Copy out your final answers or solutions in the blank spaces of the document to be sent to your tutor. It is preferable to use a pencil.

3. Include a clear and detailed solution with the answer if the problem involves several steps.

4. Mail only one homework assignment at a time. After correcting the assignment, your tutor will return it to you.

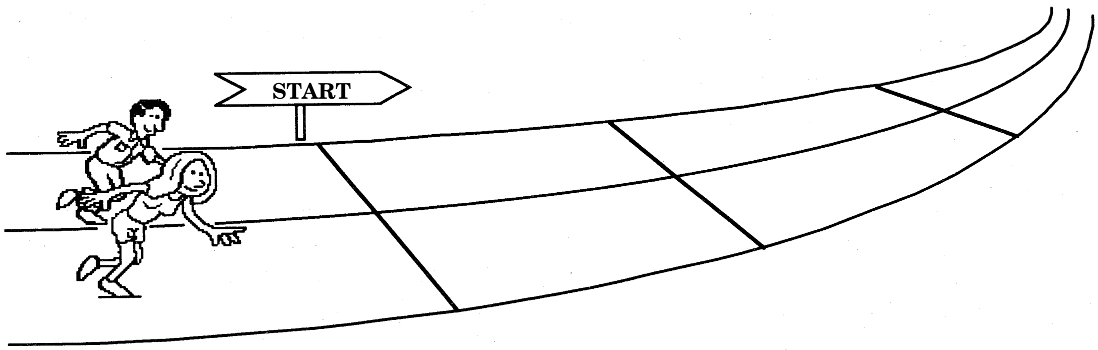
In the section "Student's Questions", write any questions which you may wish to have answered by your tutor. He or she will give you advice and guide you in your studies, if necessary.

**In this course**

Homework Assignment 1 is based on units 1 to 5.
Homework Assignment 2 is based on units 3 to 6.
Homework Assignment 3 is based on units 1 to 6.

**CERTIFICATION**

Once you have completed all the work, and provided you have maintained an average of at least 60%, you will be authorized to write the final examination.




# UNIT 1

# VALUE OF THE NUMERICAL COEFFICIENT AND THE EXPONENT OF A GIVEN BASE

## 1.1 SETTING THE CONTEXT

### Eureka!

Eureka, I've got it! This is probably what Albert Einstein said when he finally discovered how to calculate the quantity of energy generated by an object in motion. Since his discovery, we have been able to find this quantity by using the mathematical ***formula*** $E = mc^2$, where $E$ represents the quantity of energy sought. The expression $mc^2$ in this famous formula is a ***monomial***. What does this mean? Simply that we are dealing with an ***algebraic expression*** composed of only one term, a term made up of ***constants*** and ***variables*** to which only multiplication applies.

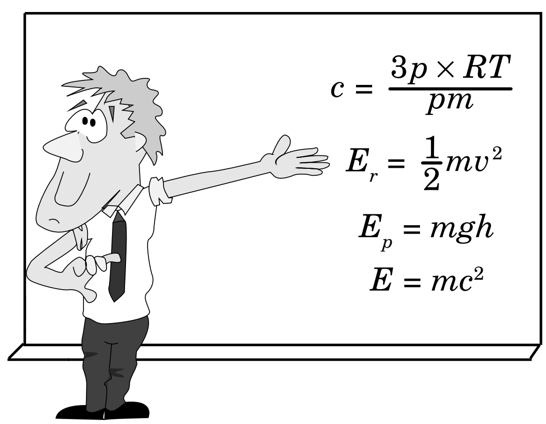
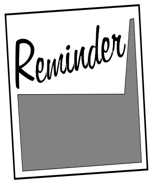



Fig. 1.1 Eureka!



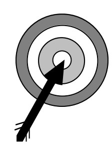

*A variable is a mathematical symbol which can be assigned any one of a set of values.*

In the algebraic expression $mc^2$, "$m$" is the variable. This letter represents the mass of the object that generated the quantity of energy sought. The mass varies according to the object involved. For example, a marble has a smaller mass than a billiard ball.

In this same expression, "$c$" is a constant, since this letter represents the speed of light in a vacuum. This speed does not vary; it is always the same. In mathematical formulas, we often use letters to represent a constant number, especially if this number is extremely large or extremely small. In the preceding example, $c$ = 299 792.5 km/s.

In this unit, we will not distinguish between the variables and the constants in an algebraic expression. We will only use the word "variable." However, it is important to know that a "letter" in an algebraic expression can represent both constant values and variable values.

**To reach the objective of this unit, you must be able to identify the *numerical coefficient* and the *exponent* of a given *base* in a monomial, and to recognize *similar monomials*.**



Let us return to the term "monomial" now to define it.

A **monomial** is an algebraic expression composed of the product of a numerical coefficient and variables.

Reminder

*A numerical coefficient is a number that multiplies the letters (variables and constants) in an algebraic expression.*

**Example 1**

a) $2t^3$ is a monomial in $t$; its ***numerical coefficient*** is 2.
b) $-\frac{1}{2}x^2y^3$ is a monomial in $x$ and $y$; its **numerical coefficient** is $-\frac{1}{2}$.

*N.B.* The monomial $-\frac{1}{2}x^2y^3$ can also be written $\frac{-x^2y^3}{2}$.

What is the numerical coefficient of $mc^2$? ..............................

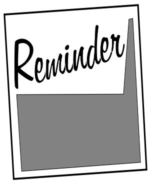
What is the numerical coefficient of the monomial $-y^4$? ..............................

What is the numerical coefficient of the monomial $-\frac{3}{4}x^3y^4$ ? ........................

In the monomial $mc^2$, the numerical coefficient is 1 because $mc^2 = 1\,mc^2$. Why? You already know that the product of any number or any ***arithmetic expression*** and 1 is equal to this number or this expression. It is therefore unnecessary to write the 1 in front of the expression or mononial above because it does not change the value of the product.

The same goes for the monomial $-y^4$. Here, the numerical coefficient of the expression is $-1$, since $-y^4 = -1y^4$.

Finally, in the monomial $-\frac{3}{4}x^3y^4$, the numerical coefficient is $-\frac{3}{4}$. Any number can be a numerical coefficient. In this module, we will use rational numbers as numerical coefficients.




*The set of rational numbers, designated $\mathbb{Q}$, is composed of natural numbers $\mathbb{N}$ = {0, 1, 2, 3 ...}, integers $\mathbb{Z}$ = {..., –2, –1, 0, 1, 2, ...}, fractions, improper fractions, mixed numbers and decimals.*

Let us return to Einstein and his formula for calculating energy. In the monomial $mc^2$, the letter "$c$" is raised to the second ***power*** (***squared***). The number 2 attached to this letter and in a raised position is called an **exponent** and we say that "$c$" is a **base** raised to the second power.

In reality, $c^2$ means $c \times c$. The exponent $^2$ indicates that the number "$c$" is multiplied by itself. In other words, the exponent attached to a quantity indicates how many times this quantity is taken as a factor. Thus 2, 5 and 7 are the respective exponents in $4^2$, $y^5$ and $x^7$.

The expression $4^2$ means $\underbrace{4 \times 4}_{\text{2 times}}$.

The expression $y^5$ means $\underbrace{y \times y \times y \times y \times y}_{\text{5 times}}$.

The expression $x^7$ means $\underbrace{x \times x \times x \times x \times x \times x \times x}_{\text{7 times}}$.

Exponents are always placed to the right of the number or letter that they raise to a given power and in a raised position relative to this letter or number. They are often written in smaller characters than the other digits.

As you can see, writing $y^5$ instead of $y \times y \times y \times y \times y$ simplifies the writing. We therefore represent the expression $y \times y \times y \times y \times y$ by its exponential form $y^5$.

**Example 2**

How would you write the following in exponential form?

a) $-2 \times (-2) \times (-2)$ b) $\frac{3}{5} \times \frac{3}{5}$ c) $\frac{7 \times 7 \times 7 \times 7}{10 \times 10}$

Answers:

a) $(-2)^3$ b) $\left(\frac{3}{5}\right)^2$ c) $\frac{7^4}{10^2}$

**Example 3**

Identify the base and the exponent in each of the following exponential forms.

a) $8^2$ b) $\left(\frac{1}{2}\right)^4$ c) $(-3)^5$

Answers:

a) The base is 8 and the exponent is 2.
b) The base is $\frac{1}{2}$ and the exponent is 4.
c) The base is $-3$ and the exponent is 5.

*N.B.* When the exponent is 2, as in the monomial $mc^2$, we say that the letter "$c$" is **squared**. If the exponent is 3, as in the monomial $2t^3$, we say that the letter "$t$" is ***cubed.*** No particular terms exist for designating exponents greater than 3.

? In your opinion, do the expressions $2^3$ and $3 \times 2$ have the same value?

................................

The answer to this question is no. In fact , $2^3$ (2 cubed ) is equal to: $2 \times 2 \times 2 = 8$. In the other expression, $3 \times 2 = 6$. For the same reason, $b^3$ is not equal to $3b$.

**Example 4**

Given the monomial $4x^2y^3$:

- In this monomial, 4 is the numerical coefficient.
- The letters $x$ and $y$ represent the given bases.
- The exponent of the base $x$ is 2.
- The exponent of the base $y$ is 3.

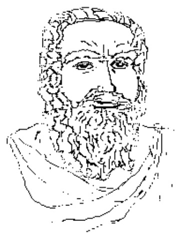
**Example 5**

Given the monomial $-\frac{3}{2}x^2y^4z^2$:

- In this monomial, $-\frac{3}{2}$ is the numerical coefficient.
- The letters $x$, $y$ and $z$ are the given bases.
- The exponent of the base $x$ is 2.
- The exponent of the base $y$ is 4.
- The exponent of the base $z$ is 2.

? Let us return to our friend Einstein. The algebraic expression $mc^2$ is a monomial as you already know. What is the exponent of the base "$m$" of this monomial? ..............................

If your answer to this question is 1: bravo! In mathematics, a number or a variable raised to the first power is equal to this number or this variable. Thus, $x^1 = x$ and $5^1 = 5$.



*Did you know that*

each one of us has around $2^{20}$ ancestors if we go back 600 years? The figure below shows the fact that each person alive today has 2 parents, 4 grandparents, 8 great-grand-parents and so forth.

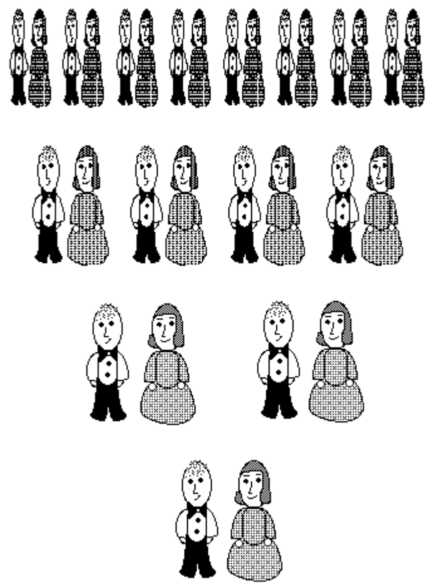


Fig. 1.2 Family portrait

In other words, if we go back one generation there are 2 ascendants. If we go back two generations, there are 4, that is, $2 \times 2$ or $2^2$. Going back three generations, there are $2 \times 2 \times 2$ or $2^3$. And going back four generations, there are $2^4$ (16) ascendants.

If we allot 30 years for each generation and calculate back 600 years or 20 generations ($600 \div 30 = 20$), the total would be $2^{20}$ ascendants or 1 048 576 ancestors! This makes an impressive genealogical tree!

## Exercise 1.1

1. Determine the value of the numerical coefficients in the monomials below.

   a) $3y^7$ ............ b) $\frac{1}{2}x^4y^3$ ............ c) $-x^7y^8$ ............

   d) $-\frac{1}{2}a^2bc^4$ ............ e) $0.12c^4y$ ............

2. What is the value of the exponent of the base $x$ in the monomials below?

   a) $-x^3y^7$ ............ b) $4x^2y^3z^4$ ............ c) $0.33y^4xz^4$ ............

   d) $-\frac{1}{2}x^8y^4$ ............ e) $3y^2z^4x^2$ ............

3. What is the exponent of the base $y$ of the monomials in Question 2?

   a) .............. b) .............. c) .............. d) .............. e) ..............

4. For each of the monomials below, identify the numerical coefficient, the exponent of the base $a$ and the exponent of the base $b$.

| | Numerical coefficient | Exponent of base $a$ | Exponent of base $b$ |
|---|---|---|---|
| a) $8a^2b$ | ...................... | ...................... | ...................... |
| b) $\frac{7}{8}ab$ | ...................... | ...................... | ...................... |
| c) $-ab^2$ | ...................... | ...................... | ...................... |
| d) $-0.47a^3b^4$ | ...................... | ...................... | ...................... |
| e) $72b^4a^9$ | ...................... | ...................... | ...................... |

Let us follow our "monomial" down the road. In mathematics, we often speak of ***similar monomials.*** What does this mean? To help you understand better, we will look at an example.

Consider the monomials $12x^3y^2z^3$, $-x^3y^2z^3$ and $5x^3y^2z^3$.

These monomials are composed of the same variables, namely, $x$, $y$ and $z$. The variables are respectively raised to the same power, that is, $x^3$, $y^2$ and $z^3$. These monomials are therefore similar monomials. Only the numerical coefficients are different.

Thus, the monomials $8t^3$ and $-4t^3$ are similar monomials since they are composed of the same variable $t$ affixed with the same exponent, namely, 3.

Now consider the monomials $8x^3y^4$, $-5x^4y^3$ and $x^5y^4$.

These monomials are composed of the same variables, namely, $x$ and $y$. However, these variables are not respectively raised to the same power since we have $x^3y^4$, $x^4y^3$ and $x^5y^4$ . They are therefore different monomials.

The monomials $7t^8$ and $9t^2$ are also different monomials since the variable $t$ is not affixed with the same exponent.

Therefore,

> Two or more **monomials** are **similar** if they are composed of the same variables respectively affixed with the same exponents, regardless of their numerical coefficients.

**Example 6**

Find the similar monomials among the following:

$$-6x^3y^8,\ 7a^2b^5,\ -3x^4y^8,\ 6a^2b^5,\ 9x^4y^8.$$

Answer:

The monomials $7a^2b^5$ and $6a^2b^5$ are similar since they are composed of the same variables respectively affixed with the same exponents.

$-3x^4y^8$ and $9x^4y^8$ are similar, but the monomial $-6x^3y^8$ is different since the variable $x$ is not raised to the fourth power as are the others.

The concept of similar monomials will be very useful to you in the following unit where you must add, subtract, multiply and divide algebraic expressions. Now go ahead and do a few exercises.

**Exercise 1.2**

1. Identify the similar monomials among the following:

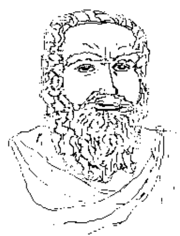
   $-6x^8y^3$, $7a^5b^2$, $-3x^8y^4$, $6a^5b^2$, $9x^8y^4$.

   ..........................................................................................................................

2. A number of the monomials in the list below are similar. Write the letters corresponding to similar monomials on each of the lines below. Be careful: there may be more than two monomials on a line.

| **List of monomials** | | | **Similar monomials** |
|---|---|---|---|
| A. $-3x^4$ | B. $-b^8$ | C. $-5x^2z^9$ | .......................................... |
| D. $0{,}6x^4$ | E. $12xy^2$ | F. $25b^8$ | .......................................... |
| G. $-4x^2z^9$ | H. $\frac{3}{16}x^4$ | I. $-11xy^2$ | .......................................... |
| J. $a^2b^3c^4$ | K. $15x^9z^2$ | L. $3a^2b^3c^4$ | .......................................... |

You have reached the end of the first unit. You now know what a monomial is and can identify the numerical coefficient of a given monomial as well as the exponent of a designated base. You are also able to recognize similar monomials. It is time to go on to some practice exercises to verify whether you have correctly understood these concepts.



*Did you know that*

exponents can "go a long way"? This guessing game for "specialists" in exponents will convince you! Suppose that you have a thin piece of paper. This paper is $\frac{1}{50}$ mm thick. In other words, you need 500 pieces to obtain a one centimetre thickness. You tear this sheet in half and place the two halves one on top of the other, then tear them again in half and place the four pieces one on top of the other, then tear the pile in two and pile the 8 pieces on top of each other, and so on.

Suppose that you stop after 50 tears. What will the height of your pile be?

The answer is 22 million kilometres! How is this possible? The first tear gives $2^1$ or 2 pieces. The second gives $2^2$ or 4 pieces. The third gives $2^3$ (8 pieces) of thickness and so on.

After fifty operations, you would therefore have $2^{50}$ pieces of paper piled on top of each other. How much is $2^{50}$? Something like 1 126 000 000 000 000. You already know that there are 500 pieces per centimetre. This means that the pile will be about 2 252 000 000 000 cm high or more than 22 million km! This is a long way for such a small piece of paper to go, isn't it?

## 1.2 PRACTICE EXERCISES

1. What is the numerical coefficient of each of the following monomials?

a) $-8x^3$ ..................... b) $x^2yz$........................ c) $\frac{2}{5}b^5$ ..........................

d) $-a^4b^2$ .................... e) $-11x^5y^3$ ................. f) $0.44x^4y^3$ .....................

g) $-0.87a^7b^3$ ............. h) $67d^4e^9$ .................... i) $1.63abc$......................

j) $-\frac{7}{8}x^8y^3$ ................

2. What is the exponent of the base $x$ in the monomials below?

a) $-7.47x^8y^7$ ............. b) $\frac{4}{3}xy^2z^3$ ................. c) $6.0x^3$ .........................

d) $-\frac{1}{2}y^4x^3$ ................ e) $-x^2z^4y^8$ ..................

3. What is the exponent of the base $y$ in the following monomials?

a) $-0.69y^8x^4z^3$ .......... b) $7.97x^8y^{12}$ ............... c) $-\frac{3}{8}y$ .........................

d) $888y^8z^8x^8$ .............. e) $4z^3y^3x^5$ ..................

4. For each of the following monomials, identify the numerical coefficient and the exponent of each of the bases.

| | Numerical coefficient | Exponent of each base | | |
|---|---|---|---|---|
| a) $2t^3$ | .................... | ............... | | |
| b) $4a^2b^6$ | .................... | ............... | .............. | |
| c) $-\frac{1}{2}abc$ | .................... | ............... | .............. | .............. |

| | | | | |
|---|---|---|---|---|
| d) $-y^3$ | .................... | ............... | | |
| e) $0.43ab^2c$ | .................... | ............... | .............. | .............. |
| f) $-0.88x^4y^9$ | .................... | ............... | .............. | |
| g) $173a^3b^2z^4$ | .................... | ............... | .............. | .............. |
| h) $-\frac{3}{2}a^9b$ | .................... | ............... | .............. | |

5. Identify the similar monomials among these monomials.

$1.43x^3y^4$; $-\frac{1}{2}a^7b^8$ ; $5xy^4$; $3.64x^4y^3$; $4a^7b^8$; $-4.43xy^4$; $87x^3z^4$; $\frac{7}{8}a^8b^7$ ;$-\frac{3}{4}xy^4$ .

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## 1.3 SUMMARY ACTIVITY

1. Define the following terms in your own words:

   a) Monomial:

   b) Numerical coefficient:

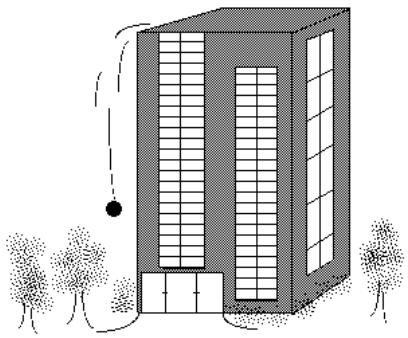
   c) Exponent:

2. Give two examples of monomials in 2 variables which have the number 3 as the numerical coefficient and a base to the third power.

3. Define the expression "similar monomials."

## 1.4 THE MATH WHIZ PAGE

**Free Fall!**



In physics, the formula for calculating the distance covered by an object in free fall is $h = \frac{1}{2}gt^2$, where $h$ is the height in metres, $t$ the time in seconds and $g$ = 9.8 m/sec$^2$, the gravitational constant.

Fig. 1.3 Watch out below!

This constant represents the acceleration of a free-falling object. This acceleration is due to the earth's force of atttraction (gravity), a force which the earth has to attract objects towards it.

This force also exists on the moon, but it is one sixth as strong as on the earth. Knowing this, we can conclude that on the moon

$$g = 9.8 \text{ m/sec}^2 \div 6 = 1.63 \text{ m/sec}^2.$$

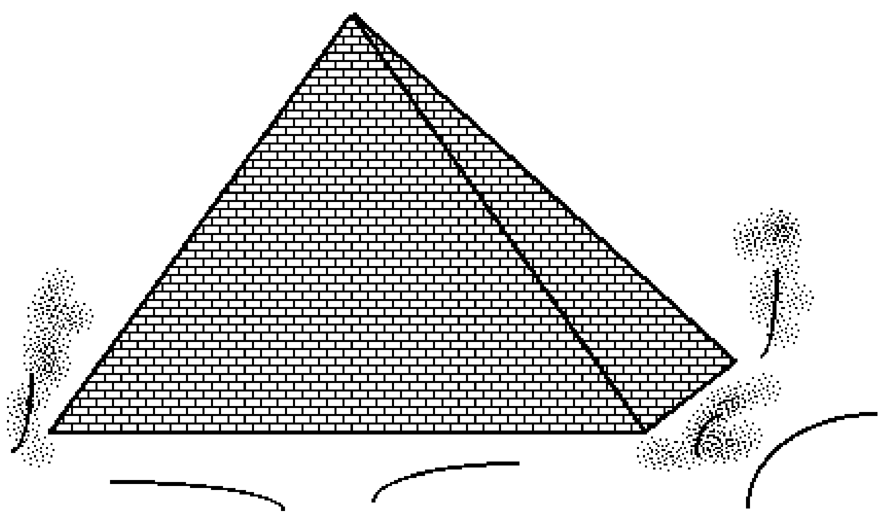
Also notice that this force is independent of the mass of the object.

It is now possible for you to calculate the height of a building or any other structure easily when you know the time it takes an object to hit the ground when it falls from the top of the structure in a free fall.

1. a) Calculate the height of a building, knowing that an object dropped from its roof takes 4 seconds to reach the ground. Give the height of this building in metres.

   b) Let us go foward in time to the year 2300. The moon has now been inhabited for 100 years and buildings are visible here and there on the moon's horizon. What is the height of a building located on the moon if an object dropped from its roof takes 4 seconds to reach the ground?

2. Using the monomial $\frac{1}{3}lwh$, we can calculate the volume of a pyramid. The height of the pyramid is represented by the letter $h$. The length and width of the base of this pyramid are symbolized by the letters $l$ and $w$ respectively. What we are looking for, the volume, is represented by the letter $V$. The formula is written $V = \frac{1}{3}lwh$.



The Great Pyramid of Egypt rises to a height of 146 m. Each side of its base measures 233 m. What volume of stone (in $m^3$) was needed to construct it?